Sensors and Materials, Vol. 34, No. 1 (2022) 367–382
MYU Tokyo

S & M 2816

Orthometric Corrections Using Gridded Gravity Data Derived from Digital Elevation Model

Hong Sool Lee, Kwang Bae Kim,* Chang Uk Woo, and Hong Sik Yun

1Department of Civil, Architectural, and Environmental System Engineering, Sungkyunkwan University, 2066 Seobu-ro, Jangan-gu, Suwon, Gyeonggi-do 16419, Republic of Korea

(Received September 27, 2021; accepted December 2, 2021; online published January 5, 2022)

Keywords: gridded gravity data, gravity, height, digital elevation model, orthometric correction

In this study, gridded gravity data (GGD) were generated by using the correlation between gravity and the height extracted from a high-resolution digital elevation model (DEM) in an area where the national control points where gravity was measured are not densely distributed. Comparison of the measured gravity and the gravity generated by using GGD, which is produced by the linear regression analysis of the height extracted from DEM at the national control points, revealed that the accuracy of GGD is about 33 mGal. The difference is very small at about 4 and 5 mm, when comparing the orthometric correction for each section and the cumulative orthometric correction calculated using the gravity interpolated from GGD with the results calculated from the measured gravity, respectively. Thus, it is considered that the orthometric correction can be effectively calculated by using the gravity interpolated from GGD at the points where there are no measured gravity data. If the gravity distribution measured at national control points becomes dense, it is expected to further improve the accuracy of the gravity interpolated from the GGD, which is generated by using the linear correlation between the gravity and the height extracted from high-resolution DEM.

1. Introduction

The height of any station on Earth is generally determined by calculating the height difference between two stations through leveling and adding the height of the station with a known height to that difference. Since height is a function in relation with gravity, it can be divided into dynamic height, normal height, and orthometric height. Dynamic height is obtained by dividing the geopotential number by a constant gravity value, γ_0, chosen as the value of normal gravity at 45° latitude. Normal height is calculated by dividing the geopotential number by the mean normal gravity along the plumb line. Orthometric height is the distance from the geoid to Earth's surface along the plumb line.[1,2] To accurately determine the height of a station, we should consider both the geometric height difference calculated by leveling and the physical height difference calculated according to the difference in gravitational potential.

*Corresponding author: e-mail: kbkim929@skku.edu
https://doi.org/10.18494/SAM.2022.3665

ISSN 0914-4935

As shown in Eq. (1), the orthometric height (*H*) can be calculated from the mean gravity ($\overline{g}$) and the geopotential number (*C*) calculated along the plumb line between the geoid and the surface station.

$$H = C / \overline{g} \tag{1}$$

Alternatively, it can be obtained by calculating an orthometric correction by using data obtained from Global Navigation Satellite System (GNSS) surveying, leveling, and gravity surveying. In flat areas, there is little change in height caused when the gravity is affected by a change in density within Earth. In mountainous areas, the horizontal planes at each point where the leveling was performed are not parallel to each other. Such a change in gravity should be considered by using the gravity value measured at each station. Therefore, to accurately determine the height of an unknown point, not only the height difference calculated by leveling, but also the orthometric correction caused by a difference in gravity should be considered.

In Korea, 70% of the land is composed of mountainous terrain, which is very uneven. In particular, the size of the orthometric correction cannot be overlooked at the leveling line crossing the mountainous areas extending in the north–south direction. By considering the orthometric correction reflecting the effect of gravity, the orthometric height must be calculated. However, in the section where leveling was performed, considering the normal orthometric correction calculated from the latitude and the average height of the leveling line, the normal orthometric height system has been maintained to this day.(3)

The formula for calculating the orthometric correction has been proposed by several geodesists. After the basic formula to obtain the orthometric correction was proposed,(1) a traditional formula for calculating it using the actual gravity was proposed.(4) A formula for calculating the orthometric correction using the Bouguer gravity anomaly (BGA) was proposed,(5) as well as a modified formula for calculating the BGA in the orthometric correction formula.(6) The orthometric correction formula was partially improved and Taiwan's orthometric correction was used for the calculation.(7,8) The advantages and disadvantages of various height systems and their practical applications are summarized in Ref. 9.

To perform an orthometric height conversion, it is necessary to review the previous studies first. It was reported that an orthometric correction of more than 10 cm was calculated at an adjacent benchmark with a height of more than 3,000 m using the newly proposed orthometric correction calculation formula in order to systematically calculate the orthometric correction in Taiwan.(7) In the Italian leveling network that crosses the Alps, the orthometric correction difference was calculated between the actual measured gravity value and the gravity value extracted from the Earth Gravitational Model 2008 (EGM2008). It was reported that the orthometric correction differences between the observed gravity and the predicted gravity from EGM2008 were 0.295 and 0.459 m in the two leveling lines AF and 155, respectively.(10) In addition, several studies to determine the rigorous orthometric heights and the practical possibility of replacing the use of Helmert orthometric heights with normal orthometric heights were carried out.(11–13)

In Korea, a precision calculation of orthometric height by orthometric correction was performed mainly through leveling and gravity surveying data measured at national control points.[14,15] Various orthometric corrections were calculated in mountainous areas by using the existing gravity data or actual measured gravity data, and then the orthometric height was determined.[16–19] The orthometric correction based on gravity data was calculated mainly for orthometric height determination in various study areas, including Seorak Mountain and Jiri Mountain. To reflect the effect of gravity due to the change in the height of the terrain, it is necessary to convert the normal orthometric height system to the orthometric height system.

Existing studies focused mainly on the calculation of the orthometric correction and orthometric height by using the measured gravity values. However, in this study, we aim to verify the accuracy of the gridded gravity data (GGD) calculated by using the correlation between height and gravity. In addition, by using the gravity value interpolated from GGD and comparing it with the orthometric correction calculated with the measured gravity value, we evaluated a change in height difference according to the undulation of the terrain in mountainous areas.

As shown in the study flow chart in Fig. 1, this study was conducted in the following order. First, after resampling from 5 m digital elevation model (DEM) to 5” DEM, the height was extracted from the national control points (benchmarks, triangulation points, and unified control points). Second, the gravity value was estimated through a linear regression analysis of the measured gravity and the height extracted from the national control points. Third, this estimated gravity value was used to generate GGD in the study area. Fourth, this GGD were interpolated into the national control points, and the difference between the interpolated gravity and the measured gravity was compared and analyzed. Thus, the accuracy of the GGD was evaluated. Finally, the orthometric correction calculated by using the gravity value interpolated from GGD

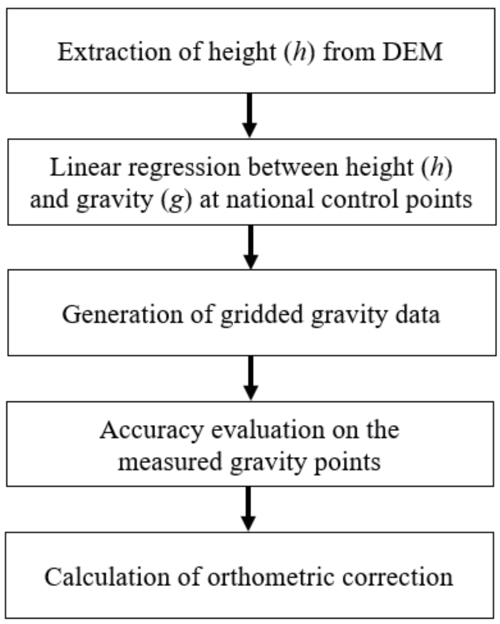

Fig. 1. Flow chart of the study.

at the national control points of the actual leveling line was compared with the orthometric correction calculated from the measured gravity.

2. Materials and Methods

2.1 Study area and data

On the basis of the high correlation between the height data and gravity data in mountainous areas, it is possible to generate GGD by using the gravity estimated by linear regression analysis and to estimate the gravity value at an arbitrary station where there are no gravity data. The study area (127.50°–128.75°E, 37.25°–38.25°N) for generating GGD includes mountainous areas with a height of 800 m or more in the Baekdudaegan Mountains in the eastern part of the Korean Peninsula, as indicated by the red rectangle in Fig. 2. As shown in Fig. 3, the gravity data measured in the study area indicated by the red square in Fig. 2 obtained a total of 1025 points in the first-order and second-order benchmarks (indicated by black and red squares) among the national control points. To ensure the distribution and homogeneity of the national control points where gravity was measured, we additionally used the gravity data measured at 538 triangulation points (indicated by black triangles) in the Seorak Mountain and Taebaek Mountain regions in 2011, 2012, and 2015 by the National Geographic Information Institute(NGII, https://www.ngii.go.kr/kor/main.do), as well as the gravity data measured at 281 unified control points (indicated

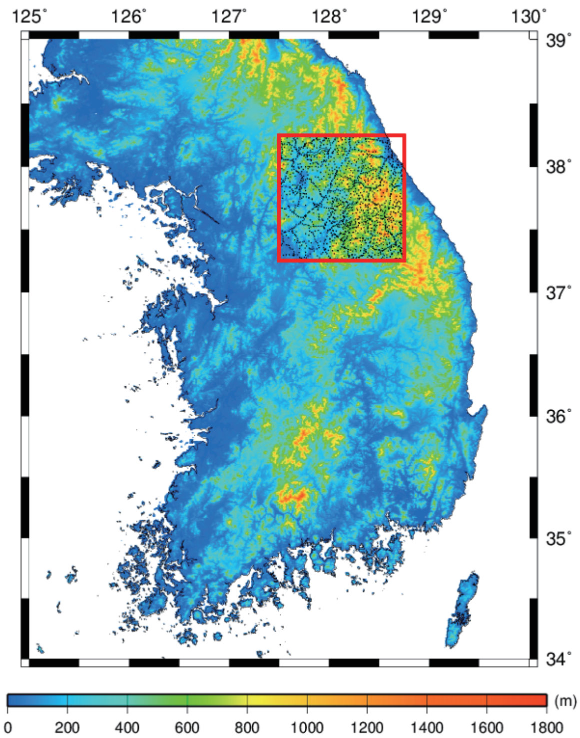

Fig. 2. (Color online) Location of study area and national control points where gravity was measured.

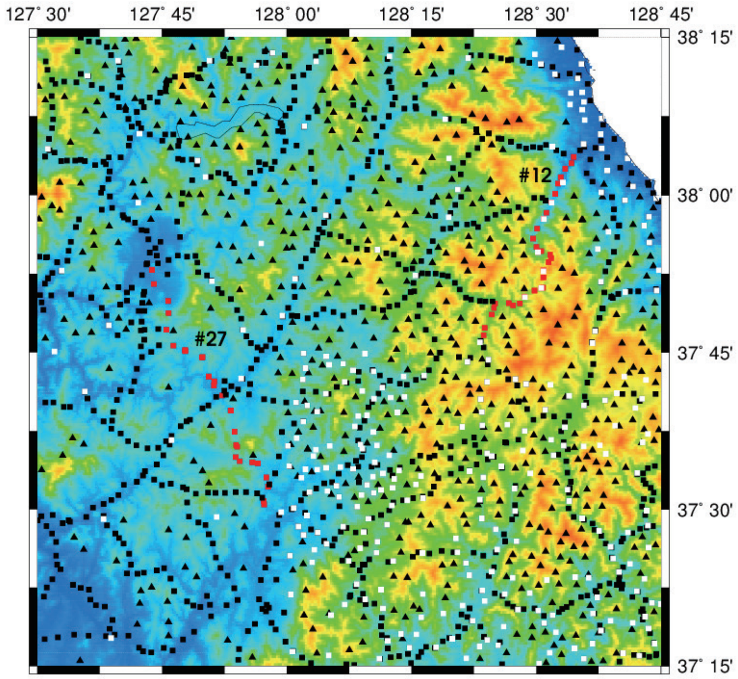

Fig. 3. (Color online) Location of measured gravity in the study area.

by white squares) in the Gangwon region in 2015 and 2018. In this study, the gravity data measured at a total of 1844 national control points (benchmarks, triangulation points, and unified control points) were used.

2.2 Calculation of orthometric correction

Orthometric height can be obtained by leveling. However, the height difference (= $H_B - H_A$) between two stations, A and B, does not coincide with the sum of geometric height differences (Δn_{AB}) obtained by leveling between stations A and B. Therefore, to obtain an accurate height difference between the two stations, it is necessary to calculate the orthometric correction (OC_{AB}) between them as shown in Eq. (2):

$$\Delta H_{AB} = H_B - H_A = \Delta n_{AB} + OC_{AB} . \tag{2}$$

Since the gravity due to the change in Earth's internal density at stations A and B can be expressed as a linear function with the height, the orthometric correction (OC_{AB}) can be calculated by using various equations.(1,4,5,7,8) Among the five equations in this study used to calculate the orthometric correction, the Heiskanen and Moritz formula was used as shown in Eq. (3) in consideration of the change in the gravity value at the two stations:

$$OC_{AB} = \sum_{A}^{B} \frac{g - \gamma_0}{\gamma_0} dH + \frac{\bar{g}_A - \gamma_0}{\gamma_0} H_A - \frac{\bar{g}_B - \gamma_0}{\gamma_0} H_B , \quad (3)$$

where γ_0 is the normal gravity at any latitude (typically 45°N or 45°S), dH is the height difference between stations A and B, g is the average of the gravity values measured at stations A and B (= $(g_A + g_B)/2$), $\bar{g}_A$ are $\bar{g}_B$ the mean gravity on the plumb line between the geoid and Earth's surface $\left(\bar{g}_A = g_A + 0.0424 H_A, \bar{g}_B = g_B + 0.0424 H_B\right)$, and H_A and H_B are the heights of stations A and B, respectively.

3. Results and Discussion

3.1 Estimation of GGD

In this study, 5" (about 150 m) gridded height data were extracted in the study area as shown in Fig. 3 by resampling a DEM with 5 m spatial resolution using the nearest neighbor method of ArcGIS Pro S/W (https://www.esri.com/). The study area includes flat land, hilly land, and mountainous area. Height interpolated from 5" gridded height data was calculated for a total of 1844 national control points (benchmarks, triangulation points, and unified control points) in the study area in order to generate GGD using the high correlation between the measured gravity and the high-resolution height data. When the extracted 5" gridded height data are interpolated to the national control points within the study area, the minimum height is 2.141 m and the maximum height is 1680.821 m. As shown in Fig. 4, the gravity data at the 1844 national control points range from 979610.715 to 980032.053 mGal. In this study, the gravity was calculated by using the linear regression equation derived from the linear correlation between the measured gravity and the height extracted from a high-resolution DEM in an area where the national control points where gravity was measured are not densely distributed. The root mean square error (RMSE) of the estimated gravity from the linear regression is 33.18 mGal.

The coefficient of determination R^2 and the coefficient of the linear regression equation were calculated in order to determine the correlation between the two data through the linear regression analysis of the measured gravity at 1844 national control points and the height interpolated from the 5" gridded height data. As shown in Fig. 4, R^2 between the height and gravity at 1844 national control points is 0.8043, thereby indicating a high correlation between the two data. As shown in Fig. 4, when the height is small, the measured gravity is large, and conversely, when the height is large, the measured gravity is small. In addition, the coefficient of the linear regression equation can be expressed as Gravity = −0.2215 × Height + 979960.9843. The height parameter estimation of the linear regression is considered to be reliable because the p-value of the F-statistic is significant within 1%.

The linear regression equation (indicated by the red line in Fig. 4) was used to estimate the gravity values for 1025 benchmarks (indicated by black and red rectangles), 538 triangulation points (indicated by black triangles), and 281 unified control points (indicated by white rectangles) among the national control points, as shown in Fig. 3 in this study. On the basis of

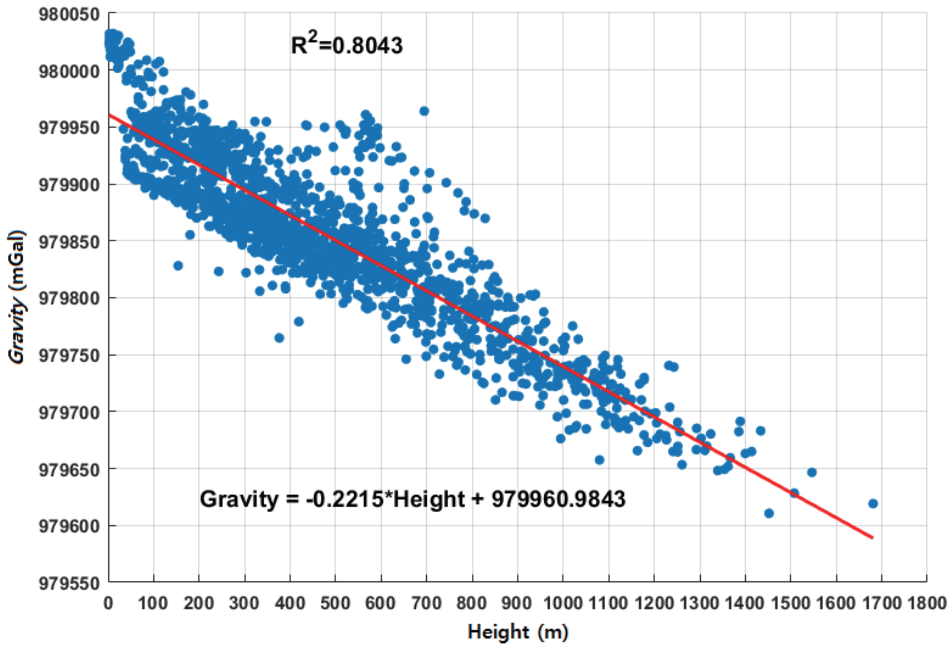

Fig. 4. (Color online) Coefficient of determination (R^2) between height and gravity determined by linear regression analysis.

these estimated gravity values, a 5" GGD of the study area was generated by using a continuous curvature surface gridding algorithm of Generic Mapping Tools (GMT, http://gmt.soest.hawaii.edu) "surface" routine.[20]

As shown in Fig. 5, 5" GGD has distributions of 979589.69 mGal (minimum), 979968.13 mGal (maximum), and 979855.75 mGal (mean). In 5" GGD of the study area, it can be seen that the gravity value in the west, where the height is small, is about 300 mGal higher than that in the mountainous area in the east with a large height, except for the sea area around the coast in the upper right area.

For the 5" GGD in Fig. 5, in order to evaluate the accuracy of the GGD compared with the measured gravity data, the gravity values were interpolated to 1844 national control points as shown in Fig. 3. Figure 6 shows the difference between the interpolated gravity from the 5" GGD and the measured gravity. As shown in Table 1, the differences are −156.70 mGal (minimum), 112.44 mGal (maximum), −0.09 mGal (mean), and 33.25 mGal (standard deviation). The accuracy according to the resolution change of the GGD was about 33 mGal, and there was no significant change. In addition, as shown in Fig. 6, the north of latitude 38° of the study area shows a gravity difference of −80 mGal or higher in the upper left area and −60 mGal or higher in the upper right area. This is an error caused by the lack of DEM data near the border between North Korea and South Korea.

Figure 7 shows a histogram of the difference between the interpolated gravity from 5" GGD and the measured gravity at 1844 national control points. At 1844 national control points within the study area, the difference in gravity is almost similar to the normal distribution curve based

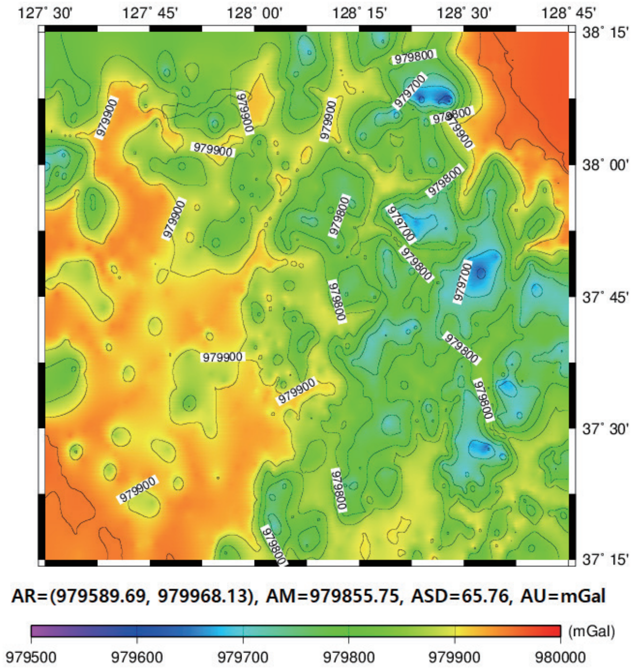

Fig. 5. (Color online) 5” gridded gravity data in the study area. The attributes listed for this and subsequent maps include the amplitude range (AR = minimum and maximum values), amplitude mean (AM), amplitude standard deviation (ASD), and amplitude unit (AU).

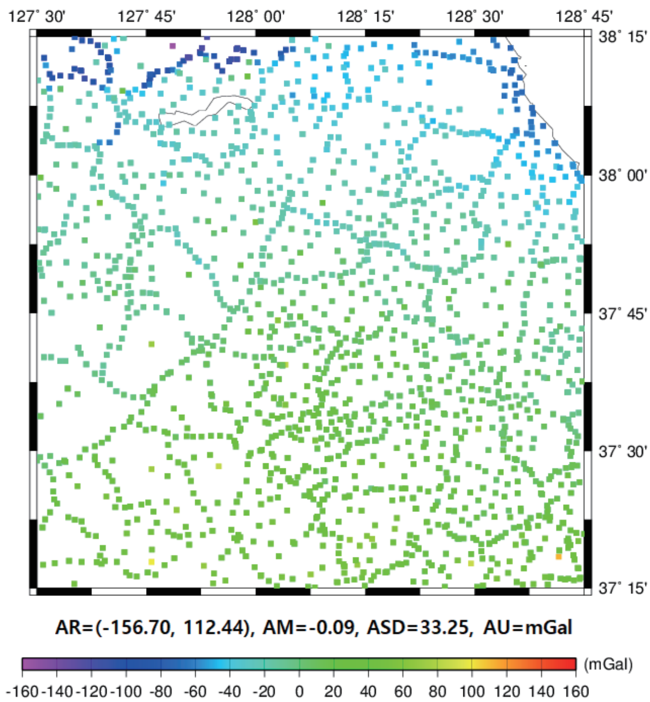

Fig. 6. (Color online) Gravity differences between the gravity interpolated from the gridded gravity data and the measured gravity at 1844 national control points.

Table 1
Statistics of differences between the gravity interpolated from the gridded gravity data and the measured gravity at 1844 national control points.

	Min. (mGal)	Max. (mGal)	Mean (mGal)	Std. Dev. (mGal)
Difference	−156.70	112.44	−0.09	33.25

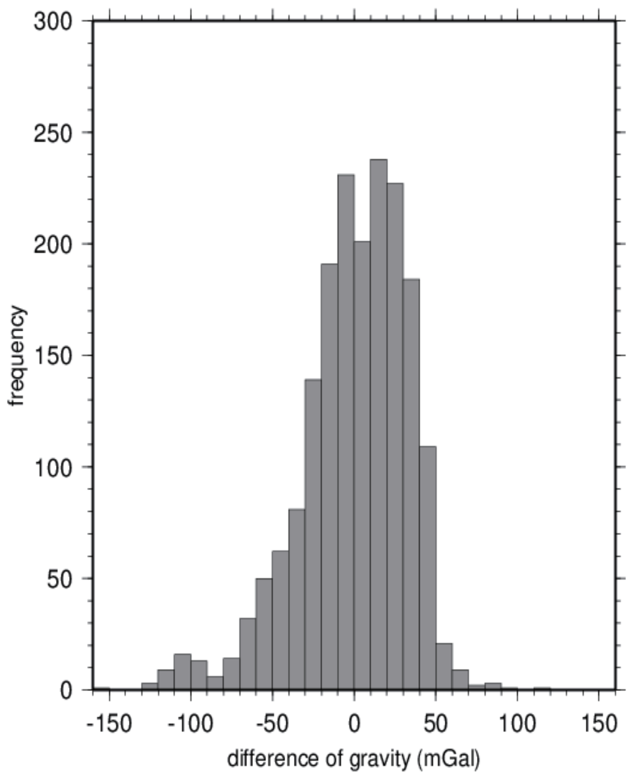

Fig. 7. Histograms of differences between the gravity interpolated from the gridded gravity data and the measured gravity at 1844 national control points.

Table 2
Percentage of differences between the gravity interpolated from the gridded gravity data and the measured gravity at 1,844 national control points.

	≤10 mGal	≤20 mGal	≤30 mGal	≤40 mGal	≤50 mGal	≤60 mGal
Difference	432 (23.4%)	861 (46.7%)	1227 (66.5%)	1492 (80.9%)	1663 (90.2%)	1734 (94.0%)

on 0 mGal. In addition, the percentage difference between the interpolated gravity from 5” GGD and the measured gravity is shown in Table 2. As shown in Table 2, the measured gravity and the gravity interpolated from GGD among the 1844 national control points show differences within ±40 mGal and ±50 mGal at 80.9% (1,492 points) and 90.2% (1,663 points), respectively. In the statistical table shown in Table 1, it was found that 1318 points (71.5%) were distributed within the standard deviation ±33.25 mGal of the difference between the gravity interpolated from the GGD and the measured gravity.

3.2 Comparison of orthometric corrections

In this study, orthometric correction using the Heiskanen and Moritz formula of Eq. (3) and cumulative orthometric correction were calculated on two routes (indicated by #27 and #12 in Fig. 3) within the study area using the measured gravity at 1844 national control points and the gravity interpolated from 5” GGD generated from the height extracted from the DEM. These results were compared with the orthometric and cumulative orthometric corrections calculated on the two routes using the actual measured gravity. In Fig. 3, the two routes #27 and #12 marked with red squares represent the first-order leveling line #27 and the second-order leveling line #12 of loop #9, respectively.

Table 3 shows the measured gravity and the gravity interpolated from 5” GGD at a total of 29 points on the first-order leveling line #27 with a maximum height of 426.0586 m. The orthometric and cumulative orthometric corrections were calculated by using the measured and

Table 3
Comparisons of gravity, orthometric correction (OC), and cumulative orthometric correction (COC) computed from the measured gravity and gridded gravity along the first-order leveling line #27.

No.	Point ID	Latitude (degree)	Longitude (degree)	Height (m)	Measured Data			Gridded Data		
					Gravity (mGal)	OC (m)	COC (m)	Gravity (mGal)	OC (m)	COC (m)
1	BM16	37.49106	127.98475	122.4464	979900.320	0.0000	0.0000	979934.248	0.0000	0.0000
2	27-01-00	37.50803	127.95461	152.6516	979895.370	0.0003	0.0003	979925.579	0.0009	0.0009
3	U_HC37	37.51360	127.95350	177.9109	979897.679	−0.0008	−0.0004	979919.849	0.0006	0.0015
4	27-02-00	37.53628	127.96231	141.7801	979901.289	−0.0001	−0.0005	979928.957	−0.0010	0.0005
5	U_HC39	37.54014	127.96490	141.5096	979902.191	−0.0001	−0.0006	979929.315	0.0000	0.0004
6	27-03-00	37.55108	127.95886	163.5545	979899.185	0.0002	−0.0005	979921.957	0.0009	0.0013
7	27-04-00	37.57364	127.94181	196.8189	979893.422	0.0005	0.0001	979917.586	0.0003	0.0016
8	U_HC36	37.57530	127.93068	218.9748	979884.437	0.0015	0.0016	979909.428	0.0013	0.0029
9	27-04-01	37.57722	127.90550	264.1510	979879.577	0.0003	0.0018	979900.401	0.0013	0.0042
10	U_HC17	37.58329	127.89741	316.9076	979870.406	0.0014	0.0032	979887.338	0.0025	0.0067
11	27-05-00	37.60017	127.90008	426.0586	979851.919	0.0035	0.0067	979866.028	0.0046	0.0113
12	U_HC18	37.60282	127.89841	400.1021	979857.285	−0.0013	0.0054	979877.523	−0.0039	0.0074
13	U_HC19	37.62393	127.89557	266.5401	979885.565	−0.0058	−0.0004	979897.675	−0.0030	0.0044
14	27-07-00	37.65781	127.88811	170.9155	979910.272	−0.0037	−0.0041	979921.928	−0.0036	0.0008
15	U_HC20	37.65791	127.88677	168.5150	979910.738	0.0000	−0.0041	979923.114	−0.0002	0.0006
16	U_HC05	37.68162	127.86997	127.8601	979925.817	−0.0018	−0.0059	979931.421	−0.0007	−0.0001
17	27-08-00	37.69747	127.85364	128.0825	979932.021	−0.0008	−0.0067	979932.644	−0.0002	−0.0003
18	U0170	37.70401	127.85425	131.8996	979932.363	−0.0001	−0.0068	979931.204	0.0001	−0.0001
19	27-09-00	37.71186	127.84350	140.2609	979932.307	−0.0001	−0.0069	979930.226	0.0000	−0.0001
20	U_HC04	37.74214	127.83023	212.5436	979920.798	0.0010	−0.0059	979914.612	0.0017	0.0016
21	27-10-00	37.74303	127.83117	207.2045	979922.005	−0.0002	−0.0061	979915.005	0.0000	0.0016
22	27-11-00	37.75419	127.79586	163.0144	979932.088	−0.0012	−0.0072	979924.051	−0.0010	0.0006
23	U0205	37.75201	127.79613	161.5999	979931.831	0.0001	−0.0072	979923.455	0.0001	0.0007
24	27-12-00	37.76081	127.77258	225.7640	979919.566	0.0013	−0.0058	979908.105	0.0020	0.0027
25	27-13-00	37.78619	127.75847	198.7964	979925.175	−0.0007	−0.0066	979914.032	−0.0008	0.0019
26	27-14-00	37.81286	127.76306	274.9533	979906.028	0.0031	−0.0035	979899.900	0.0019	0.0038
27	27-15-00	37.83194	127.76194	116.5752	979937.582	−0.0036	−0.0071	979934.591	−0.0042	−0.0005
28	U_CC08	37.85885	127.73329	81.2474	979946.499	−0.0006	−0.0077	979941.894	−0.0004	−0.0009
29	BM19	37.88117	127.72925	96.9381	979945.632	0.0000	−0.0077	979940.391	0.0000	−0.0009

interpolated gravity, respectively. When using the measured gravity, the minimum and maximum orthometric corrections for each section are −0.0058 m and 0.0035 m, respectively, and the cumulative orthometric correction is −0.0077 m at the 29th point (BM19). In addition, when using the gravity interpolated from 5” GGD, the minimum and maximum orthometric corrections for each section are −0.0042 m and 0.0046 m, respectively, and the cumulative orthometric correction is −0.0009 m at the 29th point (BM19).

Table 4 shows the measured gravity at a total of 33 points of the second-order leveling line #12 of loop #9 with a maximum height of 1011.2616 m and the gravity interpolated from 5” GGD. The orthometric and cumulative orthometric corrections were calculated by using the measured gravity and the gravity interpolated from 5” GGD, respectively. When the measured gravity is used, the minimum and maximum orthometric corrections for each section are

Table 4
Comparisons of gravity, orthometric correction (OC), and cumulative orthometric correction (COC) computed from the measured gravity and gridded gravity along the second-order leveling line #12 of loop #9.

No.	Point ID	Latitude (degree)	Longitude (degree)	Height (m)	Measured Data			Gridded Data		
					Gravity (mGal)	OC (m)	COC (m)	Gravity (mGal)	OC (m)	COC (m)
1	09-05-00-00	37.76850	128.38831	585.5846	979836.196	0.0000	0.0000	979830.827	0.0000	0.0000
2	09-05-12-01	37.77750	128.39467	577.8219	979835.911	0.0006	0.0006	979831.329	0.0001	0.0001
3	09-05-12-02	37.78942	128.39692	571.2907	979834.998	0.0009	0.0015	979826.837	0.0030	0.0031
4	09-05-12-03	37.81031	128.41078	558.4924	979843.343	−0.0042	−0.0027	979834.623	−0.0039	−0.0008
5	09-05-12-04	37.82083	128.41486	528.8628	979852.761	−0.0038	−0.0065	979843.040	−0.0033	−0.0041
6	09-05-12-05	37.82814	128.41758	536.1185	979850.781	0.0007	−0.0058	979839.879	0.0014	−0.0027
7	09-05-12-06	37.82847	128.44439	545.0409	979847.689	0.0013	−0.0045	979837.727	0.0008	−0.0019
8	09-05-12-07	37.82469	128.45347	551.3852	979846.256	0.0005	−0.0040	979839.068	−0.0011	−0.0030
9	09-05-12-08	37.82786	128.46653	576.4733	979840.624	0.0020	−0.0020	979831.611	0.0031	0.0001
10	09-05-12-09	37.83883	128.47681	602.6487	979835.239	0.0019	−0.0001	979825.633	0.0023	0.0024
11	09-05-12-10	37.84817	128.49669	625.2483	979830.862	0.0015	0.0014	979815.998	0.0048	0.0072
12	U_YK12	37.85253	128.51008	720.1167	979814.687	0.0056	0.0070	979804.308	0.0025	0.0097
13	09-05-12-11	37.85328	128.51378	756.1790	979807.600	0.0030	0.0100	979802.025	−0.0006	0.0091
14	09-05-12-12	37.86933	128.51453	928.8378	979777.294	0.0135	0.0235	979749.289	0.0327	0.0418
15	09-05-12-13	37.88069	128.51394	1011.2616	979762.626	0.0076	0.0311	979739.911	0.0024	0.0442
16	09-05-12-14	37.89383	128.52447	859.4626	979794.585	−0.0182	0.0129	979772.257	−0.0186	0.0256
17	09-05-12-15	37.90017	128.53006	771.5110	979813.549	−0.0096	0.0033	979796.088	−0.0136	0.0120
18	09-05-12-16	37.90522	128.52775	632.0485	979841.594	−0.0116	−0.0083	979823.080	−0.0109	0.0011
19	09-05-12-17	37.91097	128.50969	463.0118	979873.426	−0.0098	−0.0181	979851.734	−0.0080	−0.0069
20	U_YK11	37.90944	128.50344	358.8456	979890.687	−0.0035	−0.0216	979876.554	−0.0067	−0.0136
21	09-05-12-18	37.91742	128.49978	331.5498	979896.769	−0.0013	−0.0229	979886.125	−0.0026	−0.0162
22	09-05-12-19	37.93106	128.49319	268.3061	979911.471	−0.0029	−0.0258	979898.057	−0.0020	−0.0182
23	09-05-12-20	37.94708	128.50181	221.1875	979926.502	−0.0028	−0.0286	979906.369	−0.0011	−0.0193
24	09-05-12-21	37.96186	128.51467	187.0108	979939.218	−0.0020	−0.0306	979917.540	−0.0017	−0.0210
25	U_YK01	37.96203	128.51508	186.5220	979938.990	0.0001	−0.0305	979918.365	−0.0002	−0.0212
26	09-05-12-22	37.97206	128.52017	169.6033	979945.065	−0.0008	−0.0313	979921.193	−0.0003	−0.0215
27	09-05-12-23	37.98861	128.53006	148.9204	979955.879	−0.0015	−0.0328	979926.033	−0.0005	−0.0220
28	09-05-12-24	38.00219	128.53811	156.8521	979958.598	−0.0005	−0.0333	979920.812	0.0007	−0.0213
29	09-05-12-25	38.01856	128.54375	175.7267	979959.892	−0.0005	−0.0338	979924.154	−0.0008	−0.0221
30	09-05-12-26	38.02850	128.54900	138.0162	979970.131	−0.0011	−0.0349	979926.581	0.0001	−0.0220
31	09-05-12-27	38.04181	128.55758	63.6322	979988.572	−0.0013	−0.0362	979947.057	−0.0015	−0.0235
32	09-05-12-28	38.05019	128.57058	93.7502	979985.706	0.0000	−0.0362	979943.926	0.0001	−0.0234
33	09-08-00-00	38.06039	128.57453	38.3605	980000.754	−0.0007	−0.0369	979949.588	−0.0001	−0.0235

−0.0182 m and 0.0135 m, respectively, and the cumulative orthometric correction is −0.0369 m at the 33rd point (09-08-00-00). In addition, when using the gravity interpolated from 5” GGD, the minimum and maximum orthometric corrections for each section are −0.0186 m and 0.0327 m, respectively, and the cumulative orthometric correction is −0.0235 m at the 33rd point (09-08-00-00). The difference of COC of 1.34 cm at the 33rd point was attributable to the underestimated gravity interpolated from 5” GGD in the mountainous areas, as shown in Fig. 8(d).

On the basis of the results shown in Tables 3 and 4, it was found that when using the measured gravity at 29 points in first-order leveling line #27 with a small height and the gravity interpolated from 5” GGD, the orthometric and cumulative orthometric corrections were small. When using the measured gravity and gravity interpolated from 5” GGD for a total of 33 points on the second-order leveling line #12 of loop #9 with a relatively large height, the orthometric and cumulative orthometric corrections were found to be larger than those of the relatively small height zone.

Figure 8 shows the height at each station on the first-order leveling line #27 and second-order leveling line #12 of loop #9, the measured gravity and the gravity interpolated from the 5” GGD, and the orthometric and cumulative orthometric corrections for each section calculated by using the measured gravity and interpolated gravity from the 5” GGD. Table 5 shows the statistics on the difference between the measured gravity in the first-order leveling line #27 and second-order leveling line #12 of loop #9, and the gravity interpolated from 5” GGD, and the difference between orthometric and cumulative orthometric corrections was calculated by using the gravity and the gravity interpolated from 5” GGD, as shown in Fig. 8.

The difference between the measured gravity and the gravity interpolated from 5” GGD at the stations on the first-order leveling line #27 and second-order leveling line #12 of loop #9, and the standard deviation of the difference are 14.577 mGal and 12.651 mGal, respectively. This indicates that the standard deviation of the gravity difference for a total of 1844 national control points within the study area is smaller than 33.25 mGal. On the first-order leveling line #27 and second-order leveling line #12 of loop #9, the mean orthometric correction difference between the two points for each section was about 0.4 mm, and the standard deviation was about 4 mm. The mean cumulative orthometric correction difference for each section is about 7.5 mm, and the standard deviation is about 5 mm. However, as shown in Table 5, the maximum differences of OC of 1.92 cm and COC of 1.84 cm on the second-order leveling line #12 of loop #9 were attributable to the underestimated gravity interpolated from 5” GGD in the mountainous areas, as shown in Fig. 8(d).

These results show that in the first-order leveling line #27 with a small height and second-order leveling line #12 of loop #9 with a large height, there is no significant difference between the orthometric and cumulative orthometric corrections calculated using the gravity interpolated from the GGD generated from the linear relationship between height and gravity, and the orthometric and cumulative orthometric corrections calculated from the actual measured gravity.

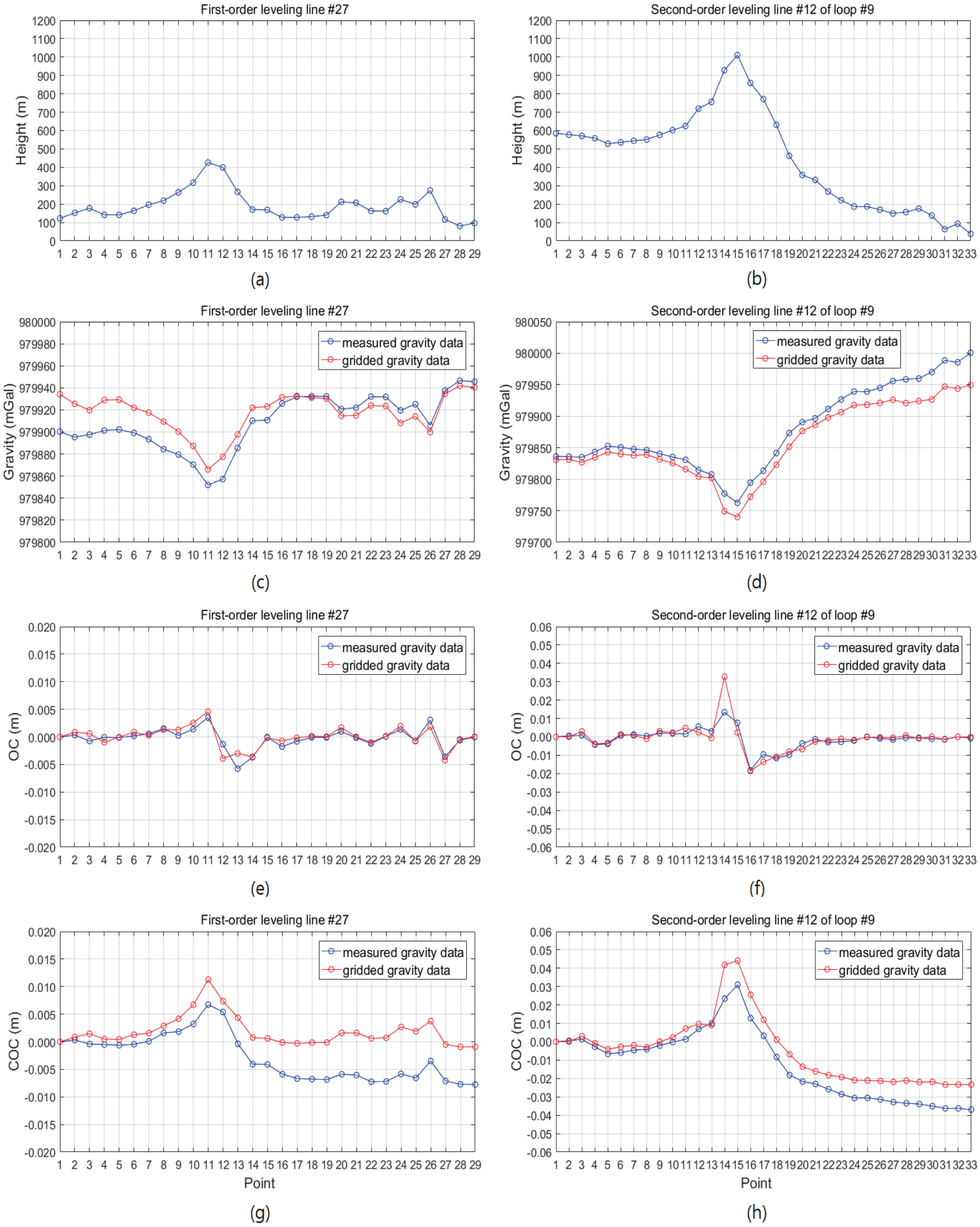

Fig. 8. (Color online) Comparisons of height, gravity, orthometric correction (OC), and cumulative orthometric correction (COC) from the measured gravity and gridded gravity in the first-order leveling line #27 (a, c, e, and g) and the second-order leveling line #12 of loop #9 (b, d, f, and h).

Table 5
Statistics of differences of gravity, orthometric correction (OC), and cumulative orthometric correction (COC) between the measured gravity and the gridded gravity along the first-order leveling line #27 and the second-order leveling line #12 of loop #9.

	Name of route	Min.	Max.	Mean	Std. Dev.
Gravity (mGal)	First-order leveling line #27	−11.461	33.928	8.727	14.577
	Second-order leveling line #12 of loop #9	−51.166	−4.582	−19.716	12.651
OC (m)	First-order leveling line #27	−0.0026	0.0028	0.0002	0.0010
	Second-order leveling line #12 of loop #9	−0.0052	0.0192	0.0004	0.0039
COC (m)	First-order leveling line #27	0.0000	0.0085	0.0047	0.0028
	Second-order leveling line #12 of loop #9	−0.0010	0.0184	0.0075	0.0051

4. Conclusions

In this study, the gridded gravity data were generated by using the high correlation and linear relationship between the height and gravity extracted from the high-resolution DEM in the areas where the national control points where gravity was measured are not densely distributed. In addition, its accuracy was evaluated by comparison with measured gravity at 1844 national control points. The following conclusions were derived.

Height data extracted from DEM are characterized by a high linear correlation with the measured gravity. In this study, the gravity was calculated by using the linear regression equation derived from the linear correlation between the measured gravity and the height extracted from a high-resolution DEM. The coefficient of determination R^2 between the two data through the linear regression analysis of the measured gravity at 1844 national control points and the height interpolated from the 5" gridded height data is 0.8043, thereby indicating a high correlation between the two data. By using this feature, GGD was generated in an area where the national control points where gravity was measured are not densely distributed. Comparison of GGD with the measured gravity at national control points revealed that the accuracy was about 33 mGal.

Calculation of the orthometric and cumulative orthometric corrections for each section along the leveling line using the interpolated gravity from the GGD revealed that the difference between the results from the interpolated gravity and the measured gravity is very small. It can be suggested that, by using the interpolated gravity at the points where there are no gravity data, the orthometric correction can be identified effectively without measuring the gravity.

If the national control points for observing gravity are more densely distributed, it is expected that the accuracy of the gravity interpolated from GGD can be further improved from the linear correlation with the height data extracted from the high-resolution DEM.

Acknowledgments

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korean government (MSIT) (NRF-2019R1C1C1005590).

References

1 W. A. Heiskanen and H. Moritz: Physical Geodesy (W.H. Freeman and Company, 1967) Chap.4.
2 P. Vaníček and E. J. Krakiwsky: Geodesy the Concepts (Elsevier, 1986) 2nd ed.
3 C. K. Lee, Y. C. Suh, B. N. Jeon, and C. H. Song: J. Korean Soc. Surv. Geod. Photogram. Cartogr. **26** (2008) 17.
4 M. M. Nassar: UNB Technical Report No. **41** (1977).
5 G. L. Strang van Hees: Zeitschrift fur Vermessungwesen **11** (1992) 727.
6 D. Dong: Formulas for Surveying and Mapping (Military Survey Book Company, 1990) p.74.
7 C. Hwang and Y.-S. Hsiao: J. Geod. **77** (2003) 279. https://doi.org/10.1007/s00190-003-0325-6
8 S.-P. Kao, R. Hsu, and F.-S. Ning: Geom. Res. Aus. **72** (2000) 43.
9 M. L. Denis and W. E. Featherstone: Proc. GG 3rd Meeting of the IGGC 2002, Ed. I. N. Tziavos (IAG, Thessaloniki, 2002) 389–394.
10 R. Barzaghi, B, Betti, D. Carrion, G. Gentile, R. Maseroli, and F. Sacerdote: Appl. Geom. **6** (2014) 17. https://doi.org/10.1007/s12518-013-0121-9
11 R. Tenzer, P. Vaníček, M. Santos, W. E. Fatherstone, and M. Kuhn: J. Geod. **79** (2005) 82. https://doi.org/10.1007/s00190-005-0445-2
12 P. A. Odera and Y. Fukuda: Earth Planets Space **67** (2015) 27. https://doi.org/10.1186/s40623-015-0194-2
13 J. O. Odumosu, O. G. Ajayi, F. F. Idowu, and E. A. Adesina: J. King Saud Univ. Eng. Sci. **30** (2018) 46. http://dx.doi.org/10.1016/j.jksues.2015.09.002
14 H. S. Yun and J. M. Cho: J. Korean Soc. Surv. Geod. Photogram. Cartogr. **22** (2004) 117.
15 Y. J. Lee, Y. S. Choi, J. H. Kwon, and J. S. Lee: J. Korean Soc. Surv. Geod. Photogram. Cartogr. **28** (2010) 99.
16 K. S. Choi and J. M. Lee: J. Korean Earth Sci. Soc. **18** (1997) 522.
17 C. Y. Kim and S. B. Lee: J. Korean Soc. Surv. Geod. Photogram. Cartogr. **29** (2011) 319. https://doi.org/10.7848/ksgpc.2011.29.3.319
18 S. B. Lee, S.-C. Auh, and C.-H. Lee: Proc. Korean Soc. Geospatial Inf. Sci. (KSGIS, 2015) 95.
19 S. B. Lee and S.-C. Auh: J. Cadastre Land Inf. **48** (2018) 99. http://doi.org/10.22640/lxsiri.2018.48.2.99
20 P. Wessel, W. H. F. Smith, R. Scharroo, J. Luis, and F. Wobbe: EOS Trans. Am. Geophys. Union **94** (2013) 409.

About the Authors

Hong Sool Lee received his B.S. and M.S. degrees from Sungkyunkwan University, Korea in 1991 and 1994, respectively. He is currently a Ph.D. candidate at Sungkyunkwan University. Since 1993, he has been working in photogrammetry and mapping, ICT, and spatial information construction. His research interests are in geodesy, GIS, and smart city. (ikaros0306@naver.com)

Kwang Bae Kim received his B.S. and M.S. degrees from Sungkyunkwan University, Korea in 1993 and 1996, respectively. He also received his M.S. degree from Ohio State University, USA in 2001 and Ph.D. degree from Kunsan National University, Korea in 2016, respectively. Since 2016, he has been a research professor at Sungkyunkwan University. His research interests are in satellite geodesy and bathymetry mapping. (kbkim929@skku.edu)

Chang Uk Woo received his B.S. degree from Sungkyunkwan University, Korea in 2021. He is currently an M.S. student of the Department of Civil, Architectural, and Environmental System Engineering at Sungkyunkwan University. His research interests are in geodesy. (dnr8874@skku.edu)

Hong Sik Yun received his B.S. and M.S. degrees from Sungkyunkwan University, Korea in 1984 and 1987, respectively. He received his Ph.D. degree from the Technical University of Budapest, Hungary in 1995. Since 1999, he has been a professor at Sungkyunkwan University. His research interests are in satellite geodesy, GNSS positioning, and geoid modeling. (yoonhs@skku.edu)