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Nationwide Level Network Adjustment of Helmert Orthometric Heights in the Republic of Korea

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In 2020, the National Geographic Information Institute (NGII) of the Republic of Korea (Korea) completed gravity observations at all vertical control points, and a basis was prepared for calculating the orthometric height. In this study, nationwide level network adjustment via the least-squares method was performed in two stages using the leveling and gravity data held by NGII, and the normal orthometric and orthometric heights at 11638 vertical control points were calculated. As a result of the nationwide level network adjustment, the orthometric height of the vertical control points in Korea ranged from a minimum of 0.5898 m to a maximum of 1325.2648 m. The difference between the normal orthometric and orthometric heights ranged from a minimum of –0.0178 m to a maximum of 0.0933 m, with an average of 0.0007 m. Therefore, the orthometric height tended to be slightly more than the normal orthometric height. After level network adjustment, the post-standard deviations of the normal orthometric and orthometric heights were identical at 2.4 mm. Because Korea has completed gravity measurements and calculated the orthometric height at all vertical control points, it is now possible to transform the height reference system to an orthometric height system.

1. Introduction

The mean sea level in Incheon Bay was defined as the Korean height datum. This is based on the results of the harmonic decomposition calculation of the tidal datum observed for approximately two years from 1943 to 1944.(1) In 1964, a level origin was installed on land, and the height of this level origin was determined to be 26.6871 m by precise spirit leveling from the mean sea level.(2,3) From this level of origin, benchmarks (BMs) were installed nationwide, with first-order BMs at 4 km intervals and second-order BMs at 2 km intervals, mostly along roads. The Korean Level Network 1987 (KLN1987) results can be considered the first modern official

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heights to be calculated by way of level network adjustment using spirit leveling data at BMs in Korea. To evaluate the performance of KLN1987, first- and second-order level network adjustments were performed separately.(4) The first-order level network adjustment targeted 439 first-order BMs, and the post-standard deviation of unit weight ($\widehat{\sigma_0}$) due to the network adjustment was 0.0049 m. The second-order level network adjustment targeted 3748 second-order BMs; as a result of the network adjustment, the post-standard deviation of unit weight ($\widehat{\sigma_0}$) was 0.0091 m.(5)

The national control points of the Republic of Korea for the survey were managed in two ways: triangulation points, which are horizontal control points, and BMs, which are height control points. Owing to the difficulty in accessing the triangulation points installed at the top of mountains and the frequent loss of the BMs installed along roads, a new solution was sought. Consequently, the National Geographic Information Institute (NGII) decided to install a unified control point (UCP), which is a three-dimensional control point, in a safe place on flat land with easy access. From this, UCP installation projects were developed. The first UCP installation project planned one point per 10 km^2 of assigned point densities, and 1196 UCPs were installed from 2008 to 2010. For the second UCP installation project, 4428 UCPs were installed from 2012 to 2019 with a point density of one point per three to five km^2.(6,7) The Korean Level Network 2014 (KLN2014) is the result of calculating the height through level network adjustment for the UCPs installed in this manner, and the post-standard deviation of the unit weight ($\widehat{\sigma_0}$) was 0.002 m.(8) The Korean Level Network 2019 (KLN2019) is the result of calculating the normal orthometric height through nationwide level network adjustment for 11481 vertical control points, which are the BMs and UCPs , and the post-standard deviation of unit weight ($\widehat{\sigma_0}$) was 0.0025 m.(9,10) Table 1 shows the history of level network adjustment in Korea.

Many countries, including the United States, Japan, and the EU nations, have changed their height reference systems from normal orthometric to orthometric height systems.(11–13) However, switching to the height reference system is undertaken if all vertical control points have gravity observation values. In 2021, NGII completed gravity observations at all BMs and UCPs and laid the foundation for calculating the orthometric height.

The purpose of this study was to determine the performance of the Korean Level Network 2022 (KLN2022), which includes not only the calculation of the normal orthometric height but also the calculation of the orthometric height by nationwide level network adjustment, targeting 11638 BMs and UCPs.

2. Materials and Methods

2.1 Spirit leveling data and gravity data

Before 2008, NGII managed a level network by connecting first- and second-order BMs, as shown in Fig. 1. There are 17 first-order level networks managed by NGII, with 16 on the mainland and one on Jeju Island. However, NGII has installed UCPs and connected them with nearby BMs continuously since 2008. Therefore, a new level of network adjustment is required. The purpose of this study is to calculate the normal orthometric and orthometric heights for all vertical control points including BMs and UCPs nationwide, so for this study, KLN2022 was

Table 1
History of level network adjustment in Korea.

Category	Main contents	Post-standard deviation ($\widehat{\sigma_0}$)
KLN1987	• Spirit leveling data at BMs from 1974 to 1986 conducted by NGII were used. • Normal orthometric height through the normal orthometric correction was calculated by level network adjustment. • First-order level network adjustment was performed in 1987. - number of leveling networks: 11, number of leveling loops: 38, number of first-order BMs: 835. - length of leveling lines: 3306 km. • Second-order level network adjustment was performed in 1988. - number of leveling networks: 15, number of leveling loops: 306, number of second-order BMs: 3748. - length of leveling lines: 8309 km.	Standard deviation of unit weight for first-order level network $\widehat{\sigma_0}$ = 0.0049 m; standard deviation of unit weight for second-order level network $\widehat{\sigma_0}$ = 0.0091 m.
KLN2006	• Normal orthometric height through the normal orthometric correction was calculated by first-order level network adjustment. - number of leveling networks: 11, number of leveling loops: 38, number of first-order BMs: 1139. - length of leveling lines: 3342 km.	$\widehat{\sigma_0}$ = 0.0018 m
KLN2014	• Normal orthometric height through the normal orthometric correction was calculated by nationwide level network adjustment. • Number of UCPs: 5371. • Mean standard error for adjusted height: 0.015 m.	$\widehat{\sigma_0}$ = 0.002 m
KLN2019	• Normal orthometric height through the normal orthometric correction was calculated by nationwide level network adjustment. - using GeoLab software fixing Incheon level origin. - number of observation data: 12572, number of control points: 11481, number of networks: 1068.	$\widehat{\sigma_0}$ = 0.0025 m

constructed. The number of first- and second-order BMs included in KLN2022 was 6180, and the number of UCPs was 5396, as shown in Fig. 2. However, the number of second-order level networks within the first-order level network continues to increase because of the continuous installation of UCPs. Thus, the number of second-order networks increased from 1068 in KLN2019 to 1489 in KLN2022. The total number of vertical control points in KLN2022 was 11638, and Table 2 shows the number of control points by category.

Leveling data for network adjustment used spirit leveling data collected by NGII from 2000 to 2021, and more detailed sections, control points, and second-order network configurations are shown in Table 3. The number of second-order networks included in the 17 first-order networks was 1489, including 1465 networks that were included in the network adjustment and 24 networks that were not included in the network adjustment. In addition, the number of sections constituting these 1489 networks included 12753 land areas and 373 island areas, and the number of vertical control points included 11312 land areas and 326 island areas, for a total of 11638 points.

Gravity data were measured using a gravity sensor mounted on a gravimeter. The gravity sensor provides a three-dimensional vector indicating the direction and magnitude of gravity.

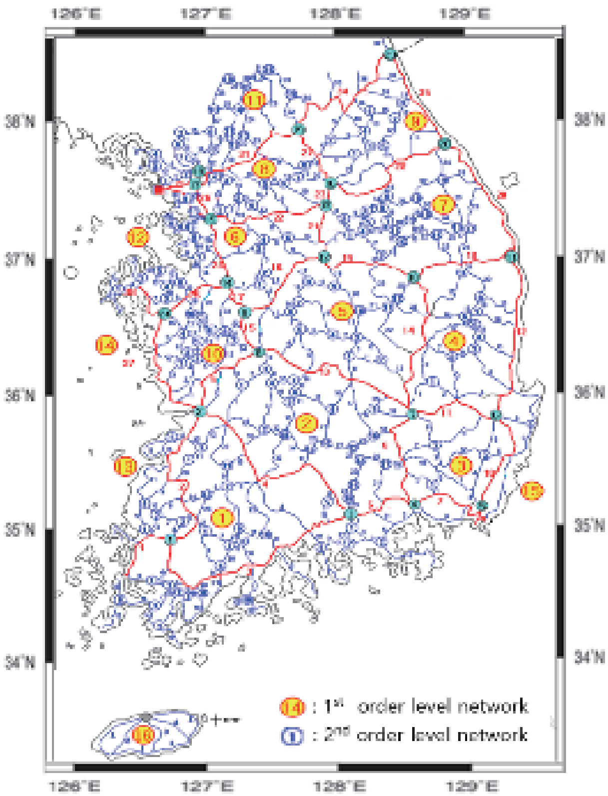

Fig. 1. (Color online) Framework of the Korean Level Network before 2008.

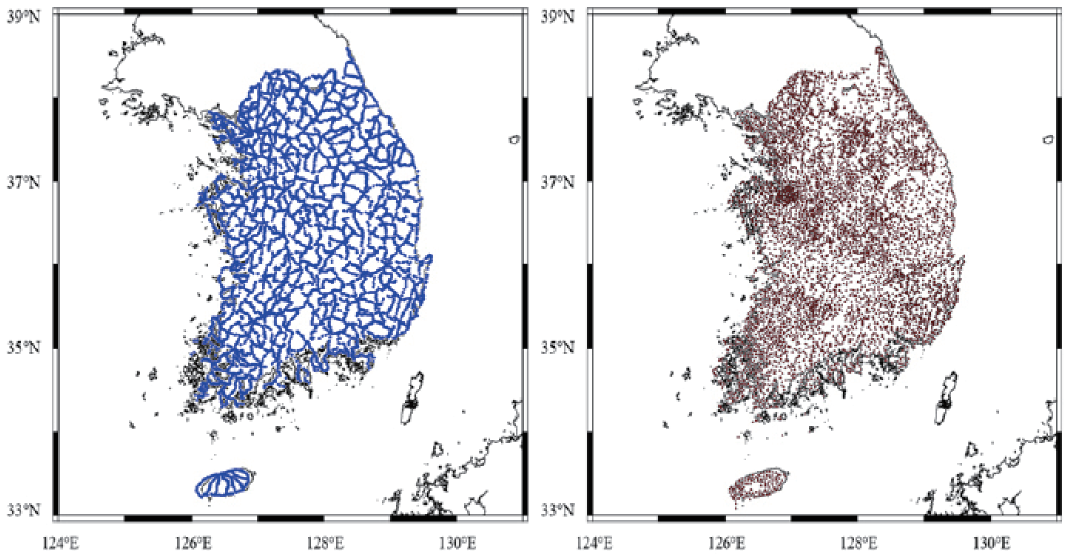

Fig. 2. (Color online) Distribution of benchmarks (left) and unified control points (right) that make up KLN2022.

Table 2
Number of vertical control points by category.

Category	Number
First- and second-order BMs	6180
Unified control points	5396
Junction BMs	22
Tidal station BMs	8
Gravity control points	18
GNSS CORS	14
Total	11638

Table 3
Composition of data used for KLN2022 nationwide level network adjustment.

Category	Number of leveling sections			Number of national control points			Number of second-order level networks		
	In the network	Opened	Sub-total	In the network	Opened	Sub-total	In adjustment	Out of adjustment	Sub-total
Land	12435	318	12753	10977	335	11312	1419	22	1441
Island	354	19	373	308	18	326	46	2	48
Total	12789	337	13126	11285	353	11638	1465	24	1489

Gravity was observed at the BMs and UCPs from 2009 to 2021, and a gravity observation database with a total of 16534 points was established by NGII. Gravity observations at the BMs and UCPs were completed in 2021. The NGII gravity database was used to calculate the orthometric correction (OC) amount for the orthometric height calculation and the distribution of gravity at the vertical control point, as shown in Fig. 3.

2.2 Orthometric height and orthometric correction

As shown in Fig. 4, the geopotential number (C) between point A_0 on the geoid and point A on the ground at point A_0 was determined using Eq. (1).[15]

$$\int_{A_0}^{A} g\, dn = W_{A_0} - W_A = C \tag{1}$$

Therefore, the orthometric height (H) can be defined using this geopotential number as

$$H = \frac{C}{\bar{g}}, \tag{2}$$

where $\bar{g}$ is the mean gravity between points A and A_0.

As shown in Fig. 4, even if spirit leveling between two points A and B on Earth is performed precisely, the exact height difference between the two points does not match the sum of the spirit

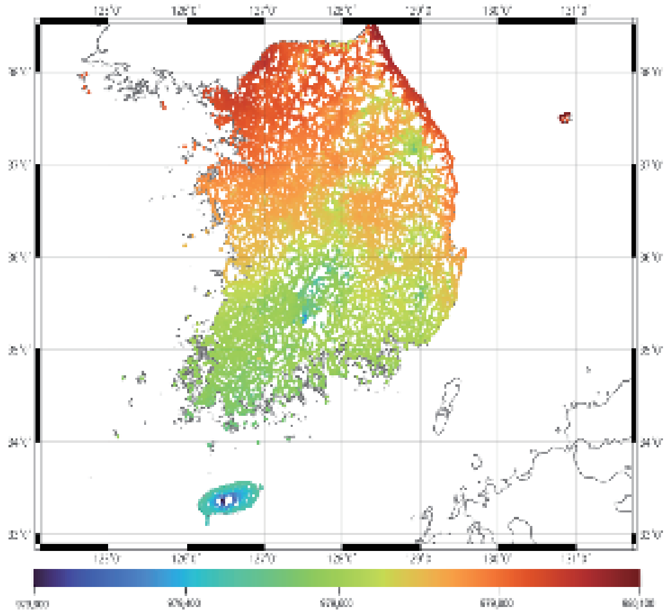

Fig. 3. (Color online) Distribution of gravity at benchmarks and unified control points that make up KLN2022.

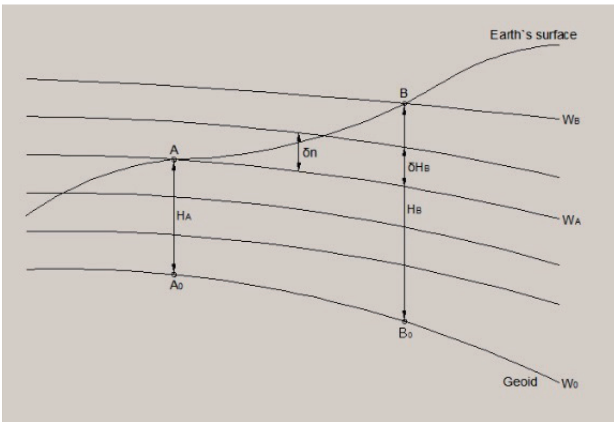

Fig. 4. (Color online) Leveling and orthometric height.[14]

leveling. The leveling result needs a correction, and the difference in orthometric height between the two points A and B requires an orthometric correction (OC_{AB}) to the spirit leveling result (Δn_{AB}) as in Eq. (3).[16,17]

$$\Delta H_{AB} = \Delta n_{AB} + OC_{AB} \tag{3}$$

The orthometric correction (OC_{AB}) between points A and B is expressed as[14,18]

$$OC_{AB} = \sum_{A}^{B} \frac{g - \gamma_0}{\gamma_0} dH + \frac{\overline{g}_A - \gamma_0}{\gamma_0} H_A - \frac{\overline{g}_B - \gamma_0}{\gamma_0} H_B , \tag{4}$$

where $\overline{g}_A$, $\overline{g}_B$ are the average gravity values of the plumbline at A and B, H_A, H_B the heights at A and B, respectively, γ_0 the normal gravity value at reference latitude (45°), and g the average gravity at points A and B.

3. Nationwide Level Network Adjustment

3.1 Formulae used for least-squares adjustment

The measurement weights included their relative values compared with other measurements and the number of corrections made to the measurements in the adjustment calculations. In the least-squares adjustment method, the correlated measurement value is proportional to the inverse matrix of the covariance matrix. The matrix obtained by dividing the covariance matrix by the reference variance of the unit weight ($\sigma_0{}^2$) is called the cofactor matrix (Q) and is represented by Eq. (5).[12,19]

$$Q = \frac{1}{\sigma_0{}^2} \Sigma \tag{5}$$

Therefore, the relationship among the weight (W), variance ($\sigma_0{}^2$), and cofactor matrix (Q) can be expressed as

$$W = Q^{-1} = \sigma_0{}^2 \Sigma^{-1} . \tag{6}$$

Even if a covariance matrix and a cofactor matrix exist in a correlated observation, the inverse matrix cannot be obtained if the cofactor matrix is a singular matrix; therefore, the weight matrix cannot be calculated.

The set of equations that define the conditions for adjustment in the adjustment computation is called the functional model. At this level of network adjustment, the parameter adjustment method was used. It constitutes an equation with the observed value, parameter, and constant for each observed value of the function model. The observation, normal, and accuracy equations are as follows.

(1) Observation equation

$$AX = L + V, W \tag{7}$$

Here, V is the residual vector, A the parameter coefficient matrix, X the parameter (start/end point height in level route) vector, L the observed value (height difference between the start and end points of the route), and W the weight of observation.

(2) Normal equation

$$\left(A^T WA\right)X = A^T WL \tag{8}$$

$$X = (A^T WA)^{-1} A^T WL \tag{9}$$

(3) Post-standard deviation of observations for unit weight (results of adjustment)

$$\widehat{\sigma_0} = \sqrt{\frac{V^T WV}{m-n}} \tag{10}$$

Here, m is the number of observations and n is the number of unknowns.

(4) Standard deviation of the adjusted height of the benchmarks and unified control points (results of adjustment)

$$M_1 = m_0\sqrt{q_{11}}, M_2 = m_0\sqrt{q_{22}}, \cdots, M_n = m_0\sqrt{q_{nn}} \tag{11}$$

Here, $Q_{(n,m)} = (A^T WA)^{-1} = \begin{bmatrix} q_{11} & q_{12} & \cdots & \cdots & q_{1m} \\ q_{21} & q_{22} & \cdots & \cdots & q_{2m} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ q_{n1} & q_{n2} & \cdots & \cdots & q_{nm} \end{bmatrix}$.

3.2 Level network adjustment

The level network adjustment was performed in two stages. In the first stage, network adjustment was performed for each second-order level loop with respect to the BMs and UCPs included in each first-order level network. In the second stage, all first-order level networks in the mainland were connected to adjust the nationwide network. Among the data accumulated by the NGII's leveling project for BMs and UCPs after 2000, the 'round trip error' and 'closed loop error' were examined using the 'Leveling Calculation Department' that constitutes each network. From this, the input data for the level network adjustment were prepared for each first-order level network.

The allowable round-trip error of the observation height difference was set to 2.5 mm $\sqrt{\mathrm{S(km)}}$ for the first-order leveling section and 5.0 mm $\sqrt{\mathrm{S(km)}}$ for the second-order leveling section by citing the 'Leveling Work Regulations' of Korea, where S is the survey distance.[20] Because the weight is inversely proportional to the square of the standard error, the weight of the first-order leveling observation was 1/section distance (km) and that of the second-order leveling observation was 1/[4 × section distance (km)]. The composition of the input data for the adjustment computation of the nationwide level network according to the first-order level network is listed in Table 4. As shown in Table 3, the section constituting the loop of the leveling section was 97.5% of the total, and that of the open section was 2.5%.

The global test is a statistic for comparing the a posteriori and a priori reference variances as a preceding process to determine whether a local test is necessary. In this study, the reason for rejecting the null hypothesis was limited to cases where there was a mistake or error in the observed values. This was corrected by determining if there were any errors in the leveling observation values used for level network adjustment. The process and method for calculating the nationwide level network adjustment shown in Fig. 5 are as follows.[21]

The nationwide level network adjustment was performed by dividing it into the first- and second-stage adjustments. In the first-stage adjustment, each first-order level network was adjusted individually. In the second-stage adjustment, the network was adjusted simultaneously across countries.

In the first-stage adjustment, a fixed point was selected as the first-order BM close to the level of origin at the top left of the network. The level network located on the land shared one or more control points with the adjacent network. However, the level network located on the southwestern coastal islands was connected by routes that do not share a control point with adjacent networks. However, independent-level networks (e.g., Jeju Island and Cheongsan

Table 4
Composition of input data for calculation of orthometric height and network adjustment according to first-order level network.

Category	No. of first-order level networks	Number of leveling sections					Number of national control points			Number of second-order level networks
		Network component		Total	Survey grade		Network component		Total	
		Loop	Opened		1st	2nd	Loop	Opened*		
Land	11	500	10	510	389	121	441	10	451	60
	12	819	16	835	680	155	704	16	720	117
	8	678	15	693	520	173	604	15	619	75
	6	690	13	703	554	149	617	13	630	74
	10	640	7	647	506	148	581	7	588	60
	5	1373	27	1400	1063	337	1199	27	1226	175
	2	1398	14	1412	1047	365	1264	14	1278	135
	7	1156	65	1221	878	343	1059	65	1124	98
	9	761	17	778	542	236	698	17	715	64
	20	804	25	829	574	255	740	25	765	66
	1	1217	24	1241	935	306	1079	24	1103	139
	4	888	10	898	650	248	794	10	804	95
	3	735	6	741	578	163	667	6	673	69
	13	592	31	623	468	155	521	31	552	73
	14	766	34	790	540	250	675	34	709	85
	15	662	4	666	484	182	609	4	613	56
	Subtotal	13679	318	13987	10408	3586	12252	318	12570	1441
	Net subtotal**	12435	318	12753	9166	3557	10977	335	11312	
Island	14 (Chungsan)	3	11	14	14	0	4	11	15	1
	16 (Jeju)	351	8	359	242	117	304	7	311	47
	subtotal	354	19	373	256	117	308	18	326	48
Total		12789	337	13126	9452	3677	11285	353	11638	1489

*Opened: routes where either the start or end point does not connect to the loop.
**Net subtotal: the data with overlapped data removed in a network adjustment.

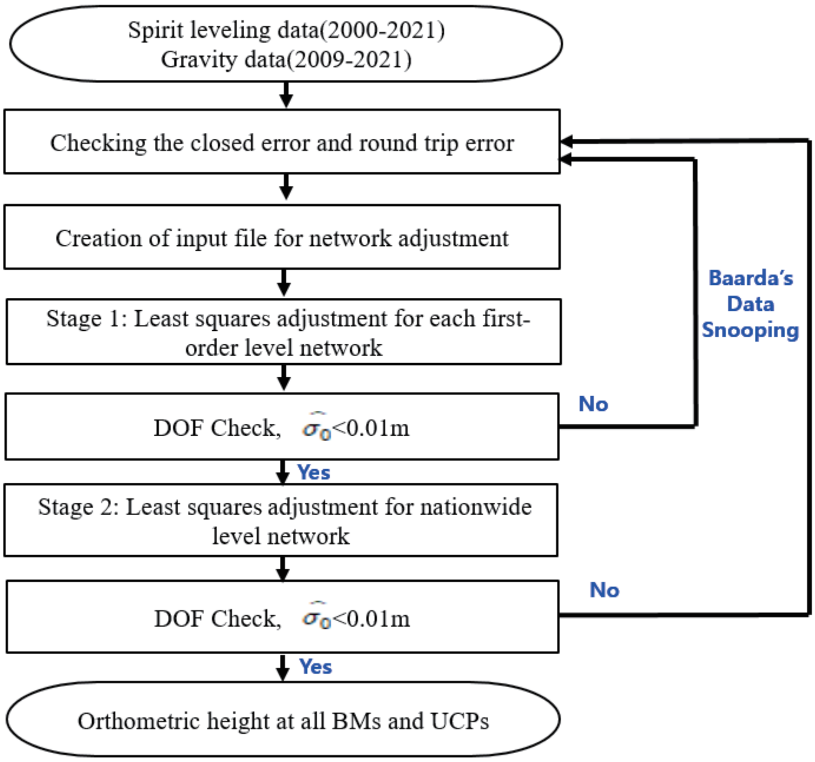

Fig. 5. (Color online) Process of nationwide level network adjustment.

Island) that were not connected to the land-level network were adjusted separately using the tidal reference point of the island as a fixed point.

For level network adjustment, 'LSAdj4LN.m', a MatLab program dedicated to the level network adjustment of NGII, was improved and used.(21) This level network adjustment program adjusts the leveling height difference of the section (start point to end point) using the least-squares method by applying a given weight. It also calculates the total number of leveling sections, the number of adjustment control points, degrees of freedom (DOFs), post-standard deviation ($\widehat{\sigma_0}$), the adjusted height of the BMs and UCPs, standard error, and standardized adjusted residual, and it saves these data as a text file.

In the primary global test, the network adjustment data were examined to determine if the number of second-order networks of input data and the DOFs (f) of the network adjustment results matched. The statistical global test was based on the χ^2 test, and a significance level α of 5% was applied. At this time, the post-standard deviation ($\widehat{\sigma_0}$) ± 0.0033 m of the 2019 research project was used as the pre-standard deviation (σ_0) of this study.

When the network-adjusted global test passed from the first second-order loop in the first-order level network, the second-order loop adjacent to the adjusted second-order loop was added, the adjustment was repeated, and all second-order loops were globally tested.

4. Results and Discussion

4.1 Results of nationwide level network adjustment

In this study, in the land area, the Helmert orthometric heights of 11312 BMs and UCPs were calculated on the basis of the Incheon level origin in this network adjustment, while in isolated islands, including Jeju Island, the BMs and UCPs deployed were calculated at 326 points on the basis of the independent tide level control point. A total of 11638 points of adjusted height were calculated by the network adjustment. Table 5 presents the results of the nationwide level network adjustment.

Table 5
Results of nationwide level network adjustment.

Category	No. of 1st-order networks	Number of leveling lines	Number of national control points	Number of 2nd-order networks	Stage 1 1st-order network adjustment*				Stage 2 1st-order cumulative adjustment**		
					Degree of freedom	$\widehat{\sigma_0}$ (m)	$\frac{f\widehat{\sigma_0}^2}{\sigma_0^2}$	χ_t^2	Degree of freedom	$\widehat{\sigma_0}$ (m)	χ_t^2
Inland	11	510	451	60	60	0.0017	16	79	60	0.0017	
	12A	824	710	115	115	0.0023	56	141	175	0.0022	
	8	693	619	75	75	0.0022	33	96	250	0.0022	
	6	703	630	74	74	0.0025	42	95	324	0.0023	
	10	647	588	60	60	0.0018	18	79	384	0.0022	
	12B	11	10	2	2	0.0021	1	6	386	0.0022	
	5	1400	1226	175	175	0.0020	63	203	561	0.0022	
	2	1412	1278	135	135	0.0025	77	163	696	0.0023	
	7	1221	1124	98	98	0.0017	26	122	794	0.0022	
	9	778	715	64	64	0.0018	19	84	858	0.0022	
	20AB	829	765	66	66	0.0016	16	86	924	0.0022	
	1	1241	1103	139	139	0.0031	123	167	1063	0.0023	
	4	898	804	95	95	0.0025	55	119	1158	0.0023	
	3	741	673	69	69	0.0022	31	89	1227	0.0023	
	13AB	623	552	73	73	0.0026	45	94	1030	0.0024	
	14ABCD	790	709	85	85	0.0034	90	107	1385	0.0024	
	15ABC	666	613	56	56	0.0033	56	74	1441	0.0024	1582
	Subtotal	13987	12570	1441	1441						
	Net subtotal***	12753	11312								
In Island	14 (Chungsan)	14	15	1	1	0.0004	0	4			
	16 (Jeju)	359	311	47	47	0.0017	12	64			
	Net subtotal	373	326	48	48	*					
	Net total	13126	11638	1489	1489	*					

*Weight of network adjustment = 1 / distance (km); 1st- and 2nd-order level surveys both apply.
**After stage 1, adjacent 1st-order level network data were added to the 1st-order level network data containing the level origin and the network adjustment was performed repeatedly.
***If the adjacent 1st-order level network adjustment data is added to the 1st-order level network adjustment data, the overlapped route data of the two networks is included twice. Thus, duplicates were excluded to include the overlapped route only once.

As a result of analyzing the precision of the level network adjustment of this study, the post-standard deviation ($\widehat{\sigma_0}$) of each first-order level network adjustment showed a distribution of at least 1.6 mm and at most 3.4 mm. However, the final post-standard deviations ($\widehat{\sigma_0}$) of nationwide level network adjustment results of the normal orthometric and orthometric heights were identical at 2.4 mm.

Table 6 shows the precisions of the level network adjustments among Korea, Japan, and North America. Korea's post-standard deviation ($\widehat{\sigma_0}$) in the 2022 level network adjustment was larger than that of Japan's 2000 level network adjustment for orthometric height transformation, which was 1.5 mm, and that of North America's 1991 level network adjustment, which was 1.2 mm. It is presumed that this was caused by artificial or natural displacement errors of the BMs owing to the long acquisition period of leveling data in Korea, uncorrected errors in leveling observation data, the insufficient systematic management of leveling observation data, and network adjustment errors.

4.2 Difference between orthometric and normal orthometric heights

The difference between the orthometric and normal orthometric heights was calculated for all BMs and UCPs used for network adjustment, and the statistical values of the differences are shown in Table 7 and Fig. 6. As shown in Table 7, because of the level network adjustment by the MatLab 'LSAdj4LevelN.m' program, the orthometric height at all vertical control points in Korea had a minimum distribution of 0.5898 m (Uyeongam 21) and a maximum of 1325.2648 m

Table 6
Comparison of Precisions of level network adjustments among Korea, Japan, and North America.

Nation	Level system	Adjustment year	Number of control points	Number of networks	Weight	Post-standard deviation ($\widehat{\sigma_0}$)(mm)
R. of Korea	Normal orthometric and orthometric heights	2022	11638	1489	1st order: 1/distance (km) 2nd order: 1/(4 × distance (km))	Normal orthometric height: 2.4 Orthometric height: 2.4
Japan*	Orthometric height	2000	16172	72	1st order: 1/distance (km)	1.4
North America**	Orthometric height	1991		71389	1st order: 1/distance (km)	1.2

*1st-order control point and 1st-order level network (excluding 14 out of a total of 86 networks).
**Canada, U.S., and Mexico connected level networks.

Table 7
Statistics of difference between orthometric and normal orthometric heights (m).

Category	Orthometric height	Normal orthometric height	Difference (between normal orthometric and orthometric heights)
Maximum	1325.2648	1325.1715	0.0933
Minimum	0.5898	0.5921	−0.0178
Average	144.2204	144.0646	0.0007
RMSE			0.0075

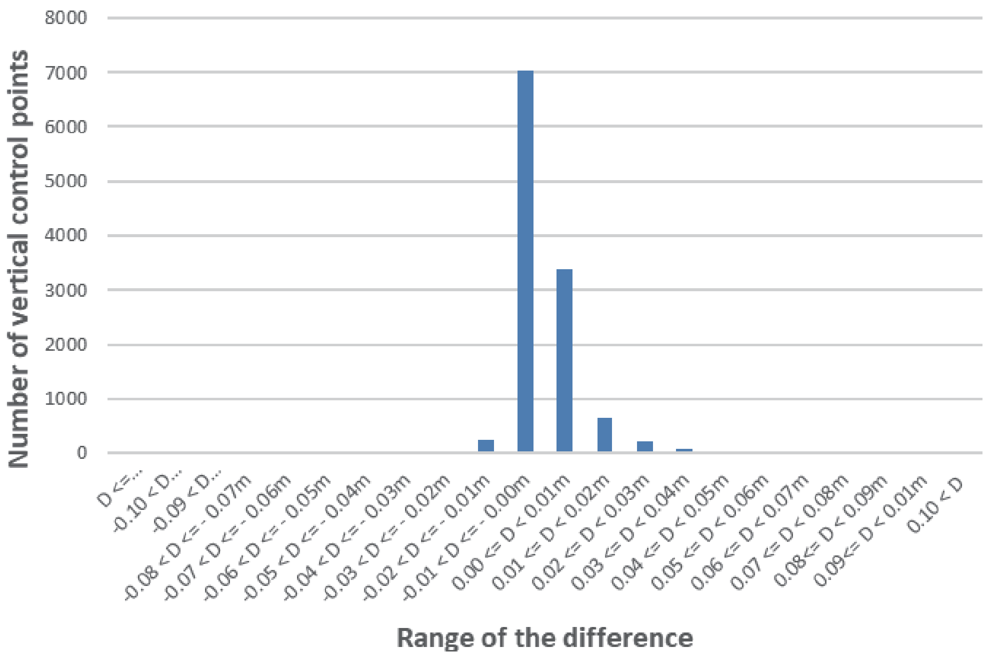

Fig. 6. (Color online) Number of vertical control points in the range of the difference.

(Utaebaek 14), with an average of 144.2204 m. The difference between the orthometric and normal orthometric heights ranged from a minimum of −0.0178 m to a maximum of 0.0933 m, with an average of 0.0007 m. Therefore, the orthometric height generally tended to be slightly larger than the normal orthometric height, and the RMSE of the difference was 0.0075 m. Also, as shown in Fig. 6, the difference between the orthometric and normal orthometric heights was 10422 points distributed within ±1 cm, accounting for 89.6% of the total. In addition, there were 22 points with a difference > +5 cm, accounting for 0.2% of the total. The point showing the largest height difference of +9.33 cm was Utaebaek 14, which had heights of 1325.1715 m based on the orthometric height and 1325.2648 m based on the normal orthometric height.

To determine the relationship between the orthometric height and the height difference, a polygonal graph was drawn for each BM and UCP in each first-order level network. The distributions of the orthometric height and height difference were similar because the means of the orthometric height and gravity values had a linear relationship in inverse proportion to each other, and because the OC was calculated in accordance with the height difference and the normal correction was calculated in accordance with the latitude value. This indicates a trend. Representative graphs of the first-order networks of Nos. 6 and 16 are shown in Figs. 7 and 8, respectively. Figure 7 shows the distribution of the orthometric height in accordance with the BMs and UCPs (top) and the variation between the orthometric and normal orthometric heights (bottom) belonging to the No. 6 first-order network, which is one of the networks representing the land area. Figure 8 shows the distribution of the orthometric height in accordance with the BMs and UCPs (top) and the variation between the orthometric and normal orthometric heights (bottom) belonging to the No. 16 first-order network, which is one of the networks representing

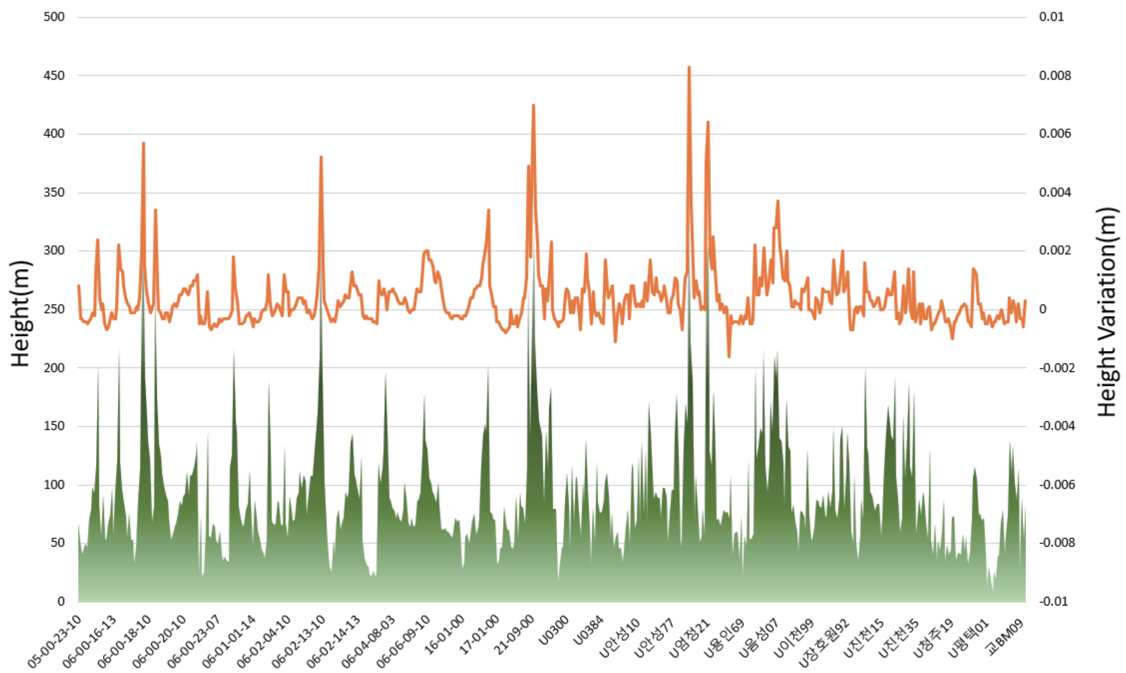

Fig. 7. (Color online) Distribution of orthometric height (bottom, green) and height variation (top, red) belonging to the No. 6 first-order level network.

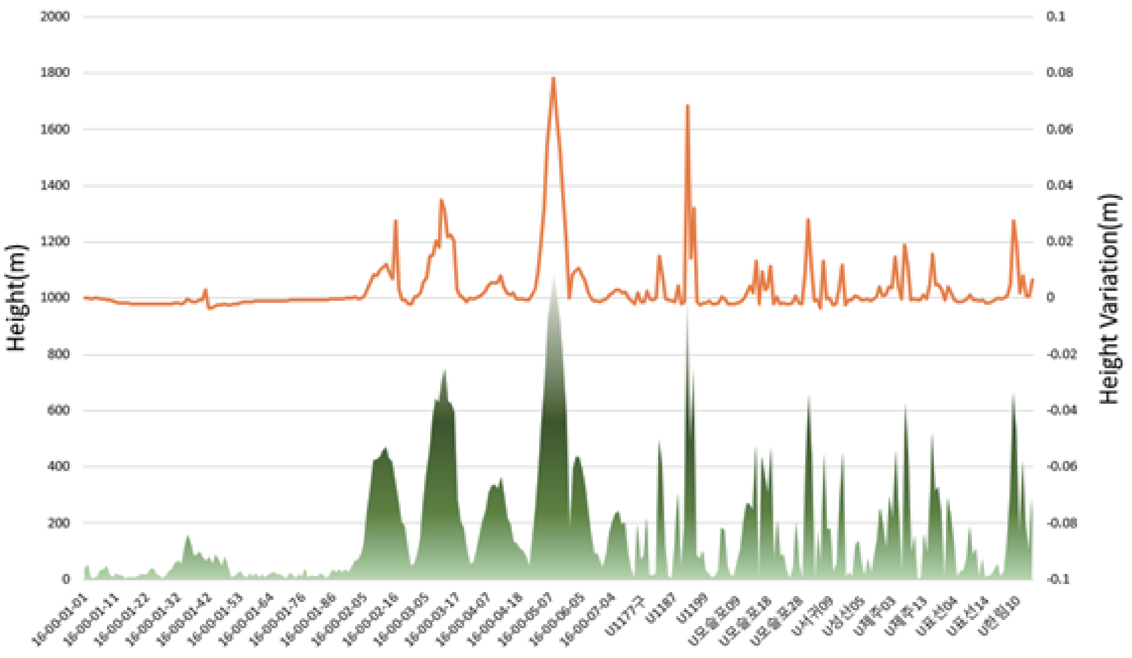

Fig. 8. (Color online) Distribution of the orthometric height (bottom, green) and height variation (top, red) belonging to the No. 16 first-order level network.

the island area. In the case of the No. 6 first-order network, the distributions of the orthometric and normal orthometric heights were 8.5880–363.1235 and 8.5866–363.1152 m, respectively. The height difference had a minimum of –0.0016 m, a maximum of 0.0083 m, and an average of 0.0004 m. In the case of the No. 16 first-order network, the orthometric and normal orthometric heights showed distributions of 2.6270–1095.5877 and 2.6277–1095.5093 m, respectively, and the difference showed a minimum of –0.0129 m, a maximum of 0.0784 m, and an average of 0.0036 m.

5. Conclusions

Many countries, including the United States, Japan, and EU nations, have changed their height reference system from a normal orthometric height system to an orthometric height system, but Korea has maintained a normal orthometric height system. However, as the NGII of Korea completed gravity measurements at all BMs and UCPs in 2021, the basis for calculating the orthometric height was established. Therefore, in this study, the nationwide level network adjustment based on the least-squares method was performed using leveling and gravity data, and as a result, the normal orthometric and orthometric heights were calculated at 11638 vertical control points, and the following conclusions were drawn.

First, because of nationwide level network adjustment for 11312 vertical control points on land, the post-standard deviations of the normal orthometric and orthometric heights were identical at 2.4 mm. This is a large value compared with the level network adjustment performance of the United States and Japan.

Second, because of nationwide level network adjustment, the orthometric heights of vertical control points in Korea ranged from a minimum of 0.5898 m to a maximum of 1325.2648 m, with an average of 144.2204 m. The difference between the normal orthometric and orthometric heights ranged from a minimum of –0.0178 m to a maximum of 0.0933 m, with an average of 0.0007 m. Therefore, the orthometric height tended to be slightly larger than the normal orthometric height.

Third, because Korea has completed gravity measurements at all vertical control points and calculated the orthometric height in this study, it is time to transform the height reference system to an orthometric height system.

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