Sensors and Materials, Vol. 35, No. 1 (2022) 307–316
MYU Tokyo

S & M 3169

Noise Model Analysis of Coordinate Time Series of Reference Station

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(Received October 31, 2022; accepted January 10, 2023)

Keywords: Continuously Operating Reference Stations, time series, noise characteristics, velocity field

To further improve the precision of the Continuously Operating Reference Stations (CORS) coordinate solution, high-precision data processing software was used to process the multi-year observation data of 13 CORS in Beijing, and the original coordinate time series and coordinate residual time series were obtained. Maximum likelihood estimation combined with power spectrum estimation was used to analyze the noise category and the best model of the CORS. The results show that the noise components in the north, east, and up (NEU) directions of the time series include white noise and flicker noise, among which the noise components in the N direction of seven CORS include random walk noise, the noise components in the E direction of one station include random walk noise, and the noise components in the U direction of three CORS include random walk noise. The best noise model of the Beijing CORS base station coordinate time series is the white noise (WN)+flicker noise (FN)+random walk noise (RWN) combined model.

1. Introduction

The Beijing Continuous Operating Reference Stations (CORS) are the core of Beijing's basic geographic information framework, which are used to establish and maintain high-precision 3D coordinate data and provide high-precision location services for real-time dynamic carrier phase users. Owing to crustal movement, social and economic activities, and other factors, the location of the Beijing CORS base station is constantly changing. To ensure timely maintenance of Beijing's basic geographic information framework, the regional base station network coordinate data is maintained and updated through time series analysis of the base station, and the spatial location, time information, and related dynamic changes of the regional base station are accurately obtained, which can provide data resources for users requiring real-time, dynamic, and high-precision positioning to ensure the stability and reliability of external services.[1,2]

The coordinate time series of a base station may include a non-long-term trend term, a step term, a noise term, and a seasonal term of amplitude that changes with time. By analyzing the nonlinear change in the time series of a base station, various geophysical phenomena and rules regarding seasonal changes can be obtained, which are helpful to improve the precision of the

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https://doi.org/10.18494/SAM4217

ISSN 0914-4935 © MYU K.K.
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base station coordinate solution.(3–5) Statistical research has shown that the coordinate time series contains time-independent white noise (WN) and time-dependent colored noise. For error analysis, not only WN but also colored noise should be considered. Based on the observation data of the Beijing CORS base station in recent years,(6–8) in this paper we analyze the best noise model of the time series.

2. Noise Analysis Method of Coordinate Time Series

2.1 Maximum likelihood estimation method

Maximum likelihood estimation is a nonlinear least squares method that can simultaneously obtain all the parameters of a time-related noise model. It can not only facilitate the discovery of the noise contained in a column of random data, but also accurately calculate the power spectrum of the noise and find the parameter model closest to the time series.(9) The likelihood function used for maximum likelihood estimation is

$$l\left(\bar{v},C\right)=\frac{1}{\left(2\pi\right)N/2\left(\det C\right)1/2}\exp\left(-0.5\bar{v}^{T}C^{-1}\bar{v}\right), \tag{1}$$

where C is the covariance matrix of the assumed noise, N is the length of the time series, and $\bar{v}$ is the linear fitting residual.

The maximum l value is equal to the maximum value of its logarithm.

$$\ln\left[l\left(\bar{v},C\right)\right]=-0.5\left[\ln\left(\det C\right)+\bar{v}^{T}C^{-1}\bar{v}+N\ln\left(2\pi\right)\right] \tag{2}$$

By adjusting the covariance matrix to maximize the likelihood function, the noise model closest to the time series can be obtained.

2.2 Power spectrum estimation

The noise of many natural geophysical phenomena has a power law property, i.e., there is a power relationship between the power spectral density of the noise and its corresponding frequency. The noise in the time series of a base station also conforms to the power law property, and the following power relationship between the noise frequencies holds:

$$P\left(f\right)=P_0 f^{\alpha}, \tag{3}$$

where $P(f)$ is the power spectral density, P_0 is the normalization constant, f is the frequency of the noise, and α is the spectral index.

The spectral index of noise in nature has a certain range: when $-3 < \alpha < -1$, we call it fractal Brownian motion, and when $-1 < \alpha < 1$, we call it fractal WN. Values of α of 0, −1, and −2 correspond to WN, flicker noise (FN), and random walk noise (RWN), respectively, and FN and RWN are colored noises. The properties of noise can be determined by estimating the power spectral density and spectral index.(1,10)

3. Acquisition of Coordinate Time Series of Base Stations

We select the observation data of the 13 CORS (BISM, BJTZ, CHAO, DSQI, MYUN, NLSH, PING, SHIJ, THKO, XIJI, XNJC, YQSH, ZHAI) in Beijing from January 1, 2016 to December 31, 2018. The synchronous observation data of eight IGS stations (BJFS, GMSD, CHAN, LHAZ, SHAO, TIXI, ULAB, URUM) are introduced for joint calculation, and the coordinate time series information of the 13 CORS is obtained.

3.1 Data processing

We process the observation data of the 13 CORS in Beijing using GAMIT 10.5 software. Because the receiver model and antenna signal of the observation data are not accurate when stored and the sampling interval is 15 s, the file header information must be standardized, and the sampling interval is modified to 30 s. After preprocessing, a baseline calculation is performed on all CORS to obtain the relaxation solution, and then the regional network is adjusted using GLOBK under the ITRF2008 framework to obtain the original time series information of the Beijing CORS from the results of the adjustment. The baseline solution strategy is shown in Table 1.

Table 1
Baseline solution strategy of GAMIT.

Parameter name	Method model
Baseline Treatment Type	Relaxation Solution
Observations	Combined Observations of Ionosphere
Reference Frame	ITRF2008
Satellite Orbit	IGS Precise Ephemeris
Satellite Cut-off Altitude Angle	5°
Data Sampling Interval	30 s
Tropospheric Correction Model	Saastamoinen
Number of Zenith Delay Correction Parameters	13
Light Pressure Model	BERNE
Solid Tide Model	IERS2010
Ocean Tide Model	FES2004
Coordinate Constraint	IGS Station: N: 0.05 m, E: 0.05 m, U: 0.1 m CORS: N: 10.0 m, E: 10.0 m, U: 10.0 m

3.2 Accuracy assessment

3.2.1 NRMS value of synchronous ring

GAMIT software is used to check the synchronization loop after the baseline solution. Generally, the normalized root mean square (NRMS) value in the Q file of the solution result is used as an indicator of the quality of the solution, which in turn is used to indicate the deviation of the baseline value calculated in a single period from its weighted average. Normally, the NRMS value is required to be less than 0.3 weeks.(11–13) The NRMS value of the synchronization ring calculated for the baseline is shown in Fig. 1. Three years of observation data have been calculated in this study, and the NRMS values of the baseline solution are all less than 0.2. The results show that the accuracy of the baseline solution results is high and meets the requirements of the baseline network adjustment. It is also found that the fluctuations of the NRMS values of the synchronization loop of the baseline solution show periodicity.

3.2.2 Repeated baseline inspection

The repeatability of each baseline component and side length is calculated after baseline processing of the base station. The repeatability of the solution vector in each period reflects the dispersion of the observation data, represents the internal accuracy of the baseline solution, and is one of the important indicators of the quality of a baseline solution. For the repeated observation of a baseline in multiple periods, if there is no gross error, the observation data should be close to the same value. The smaller the value, the higher the internal coincidence accuracy of the baseline solution. At the same time, the repeatability of each baseline side length, north–south component, east–west component, and vertical component is fitted with a straight line with a fixed error and a proportional error. As a reference index to measure the accuracy of the baseline solution, the smaller the fixed error and proportional error coefficients of the overall repeatability of each baseline component, the better the baseline repeatability and the higher the overall solution accuracy.(10,11) The baseline repeatability statistics of the base

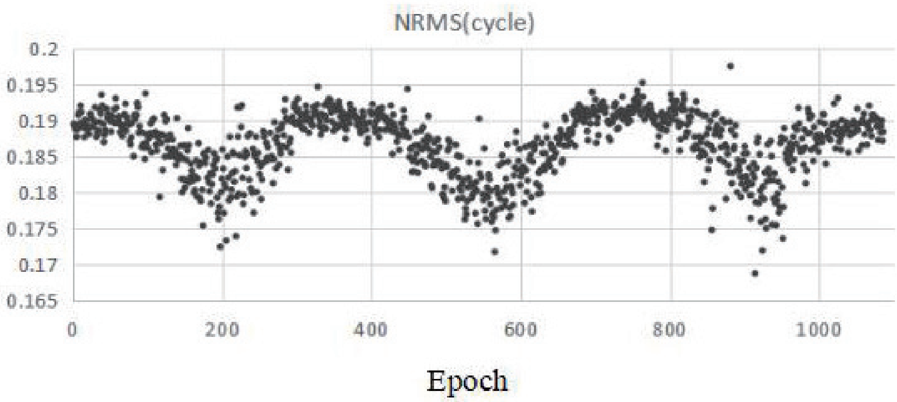

Fig. 1. Synchronization loop NRMS value for baseline solution.

station network are shown in Table 2, where RNc is the repeatability value of the baseline components for the north direction, REc is the repeatability value of the baseline components for the east direction, RUc is the repeatability value of the baseline components for the up direction, and RLc is the repeatability value of the side lengths. The baseline repeatability values of the base station network obtained by straight-line fitting are shown in Table 3.

According to the analysis in Tables 2 and 3, the average repeatability of the east–west and length components of the baseline is about 5 mm, and the fixed error and proportional error are also small. The baseline repeatability is high. The repeatability of the north–south component is inferior to that of the east–west component. The reasons for these results are as follows. Most of the CORS in Beijing are rooftop stations, and the subsidence of some stations is also serious, which reduces the repeatability in the elevation direction. The time series in the elevation direction presents a linear trend without obvious periodic characteristics.

4. Noise Analysis of Time Series of Reference Station Coordinates

4.1 Acquisition of residual time series

Before the noise analysis of the time series, it is necessary to remove the linear and periodic terms from the original coordinate time series to obtain the coordinate residual time series. The original coordinate time series can be expressed as

$$\begin{aligned} y(t_i) &= a + bt_i - c\sin(2\pi t_i) - d\cos(2\pi t_i) \\ &-e\sin(4\pi t_i) - f\cos(4\pi t_i) - \sum_{j=1}^{n} g_i H(t_i - T_{gj}) + v_i, \end{aligned} \tag{4}$$

where $y(t_i)$ is the original coordinate time series, v_i is the observation residual, t_i is the epoch time, a is the initial coordinate of the reference station, b is the linear velocity, c and d are annual periodic motion parameters, and e and f are semi-annual periodic motion parameters. $\sum_{j=1}^{n} g_i H(t_i - T_{gj})$ is the step offset caused by changes to the antenna, station site migration,

Table 2
Baseline repeatability statistics of baseline network.

Statistical value	RNc (mm)	REc (mm)	RUc (mm)	RLc (mm)
Average value	4.135	5.165	16.453	4.595

Table 3
Baseline repeatability values obtained by straight-line fitting.

Statistical content	Fixed error (mm)	Proportional error
N direction	2.418	1.4939×10^{-9}
E direction	3.603	1.3295×10^{-9}
U direction	17.335	-1.0938×10^{-9}
L direction	3.043	1.3465×10^{-9}

earthquakes, and other reasons, which can be corrected in the GAMIT data processing, and this term can be ignored. Taking the original coordinate time series as the input observation data, the least squares adjustment is performed to calculate *a*–*f* and thus obtain the coordinate residual time series. The time series of the NEU coordinate residual for the BISM station is shown in Fig. 2 as an example.

4.2 Analysis of noise characteristics of coordinate time series

In recent years, research on the coordinate time series of GNSS stations has shown that the coordinate time series includes not only WN unrelated to time, but also colored noise related to time, such as FN, RWN, and so forth. The spectral index of noise can be calculated by a power spectrum estimation method, so as to judge the general type of noise. In this study, CAST software is used to obtain the spectral indexes of the coordinate time series of the 13 CORS, as shown in Table 4. From the results of the power spectrum estimation method introduced in Sect.

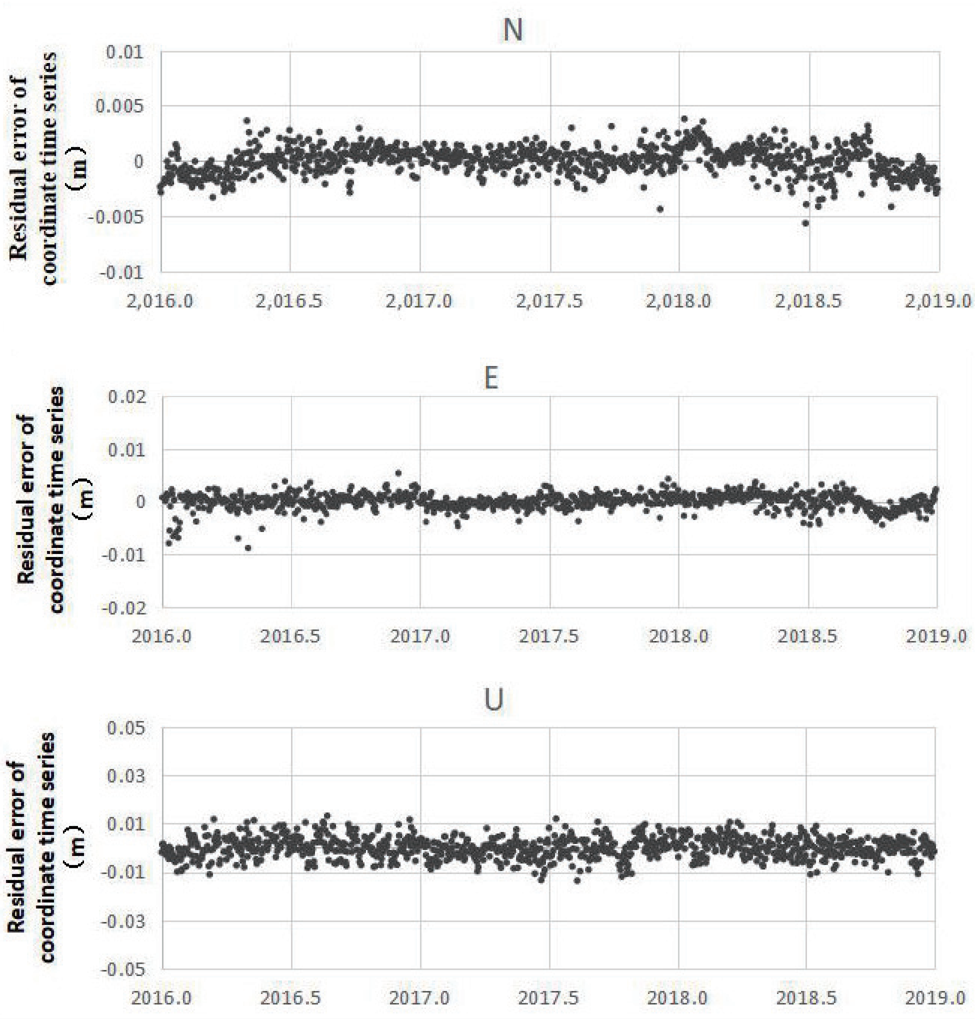

Fig. 2. Coordinate residual time series of BISM station.

Table 4
Baseline repeatability obtained by straight-line fitting.

	N	E	U
BISM	−0.6763	−0.6279	−0.6375
BJTZ	−0.9266	−0.6155	−0.4878
CHAO	−0.7631	−0.6448	−0.6143
DSQI	−0.8377	−0.6602	−0.7802
MYUN	−0.6307	−0.6171	−0.5361
NLSH	−0.8706	−0.6800	−0.9374
PING	−0.6452	−0.7399	−0.8988
SHIJ	−0.6512	−0.6611	−0.5585
THKO	−0.5962	−0.7553	−0.6589
XIJI	−0.6294	−0.6402	−0.4417
XNJC	−0.7842	−0.6834	−0.6811
YQSH	0.4276	−0.5950	−0.5421
ZHAI	−0.5480	−0.6240	−0.4611

2.2 and the calculated values of the spectral index in Table 4, we can see that the spectral index values in the NEU direction of the reference station coordinate time series are all non-integer values between –1 and 0, which indicates that the noise models of the 13 CORS are not of single noises but are models combining several types of noise.

It is assumed that the time series of the Beijing CORS is composed of two or three noise models of WN, FN, and RWN. To determine the best noise combination model, we consider the WN+FN model, WN+RWN model, and WN+FN+RWN model. Using CAST software, we estimate the maximum likelihood values of these three noise combination models and calculate the difference from the maximum likelihood values under the single monologue noise model to analyze the existence of colored noise. The results in the NEU directions are shown in Figs. 3–5.

From the analysis of Figs. 3–5, we can see that the difference between the maximum likelihood values of the three noise models and the single monologue noise model is greater than zero, indicating that there is not only WN but also colored noise in the noise of the coordinate time series. The maximum likelihood difference between the WN+FN model and the WN+FN+RWN model is greater than that of the WN+RWN model, indicating that the best noise model is the WN+FN model or the WN+FN+RWN model.(14,15) According to the Monte Carlo experiment, if the maximum likelihood difference between two noise combination models is greater than a threshold value of 2.9, it is considered that the models can be distinguished significantly. It can be seen from Figs. 3–5 that the maximum likelihood difference between the two combination models is less than the threshold value and cannot be distinguished. Therefore, we assume that the noise combination model is the WN+FN+RWN model, estimate the noise component under the combination model, and judge whether there is random walk noise. The results are shown in Table 5.

It can be seen from Table 5 that the noise components in the time series in the NEU directions of all CORS include WN and FN, the noise components in the N direction of seven CORS include RWN, the noise components in the E direction of one station include RWN (DSQI station), and the noise components in the U direction of three CORS include RWN. The noise

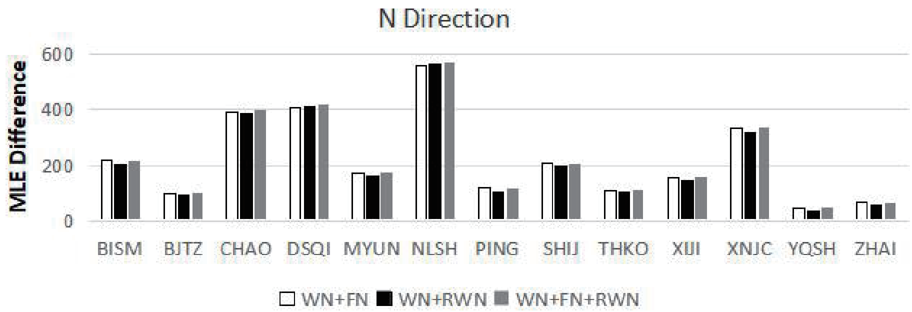

Fig. 3. Maximum likelihood difference in the N direction of each station.

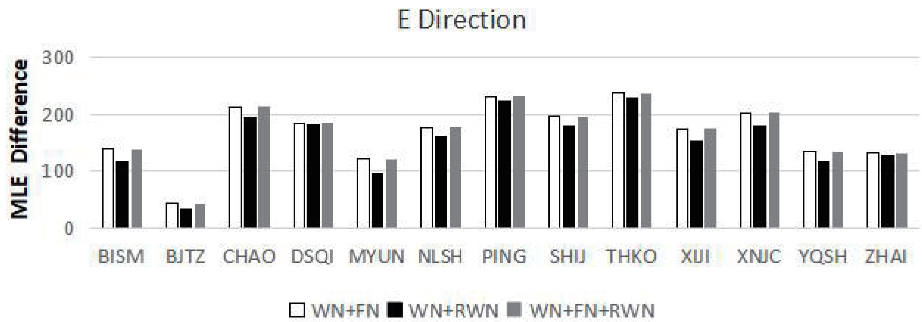

Fig. 4. Maximum likelihood difference in the E direction of each station.

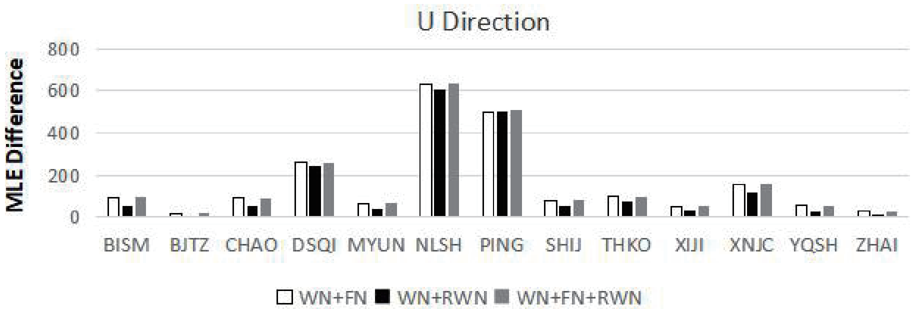

Fig. 5. Maximum likelihood difference in the U direction of each station.

level in the U direction is clearly higher than that in the horizontal direction, which is consistent with our conclusion that the positioning accuracy in the vertical direction is lower than that in the horizontal direction. The noise level in the E direction is generally higher than that in the N direction, and the FN level is greater than the WN level. The results show that the best noise model of the CORS coordinate time series is the WN+FN+RWN combined model.

Table 5
Estimation of noise component of WN+FN+RWN combined model.

Station	Direction	WN	FN	RWN	Station	Direction	WN	FN	RWN
BISM	N	0.7284	2.7321	0.8778	SHIJ	N	0.9578	2.9767	1.3433
	E	0.8931	3.6671	0		E	1.1273	4.4047	0
	U	2.3816	12.8767	0		U	3.6139	13.4309	0
BJTZ	N	0.3539	4.7809	0	THKO	N	0.8194	2.8029	0
	E	0.7719	3.3236	0		E	0.7246	3.6693	0
	U	2.7285	9.1537	0		U	2.6265	13.2481	0
CHAO	N	0.7321	2.5499	1.411	XIJI	N	0.8173	3.0969	0
	E	0.9117	3.3731	0		E	0.8831	3.5774	0
	U	2.5805	13.0079	0		U	3.7566	10.4653	0
DSQI	N	0.7314	2.3244	2.1753	XNJC	N	0.5974	3.0139	1.0612
	E	0.8821	2.5172	1.9186		E	0.7888	3.5337	0
	U	2.682	11.7635	5.5138		U	2.2224	12.7562	0
MYUN	N	0.9574	3.0334	1.1237	YQSH	N	0.9986	2.4509	0
	E	0.9746	4.0064	0		E	0.996	3.4821	0
	U	3.0362	10.8503	0		U	2.6505	10.8658	0
NLSH	N	0.7465	2.5862	2.3519	ZHAI	N	1.0588	3.3602	0
	E	0.8466	3.7731	0		E	0.9238	3.1724	0
	U	1.9757	13.4071	6.1639		U	3.3946	10.6597	0
PING	N	0.7505	3.2244	0					
	E	0.7441	3.4943	0					
	U	2.9901	11.776	9.8465					

5. Conclusion

The CORS station network is an important part of Beijing's basic geographic information framework. Time series analysis of the CORS base stations is an important way to maintain the regional reference framework and obtain the dynamic changes in base stations. In this paper, the observation data of 13 CORS in recent years are processed, the coordinate time series of the base station are obtained, and the best noise model of the coordinate time series of the base station is determined to be the WN+FN+RWN combined model, which is important for analyzing the characteristics of dynamic changes in the CORS.

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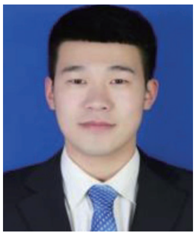

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